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NONLINEAR WAVES

# Parameterization of run-up characteristics for long bell-shaped solitary waves propagating in a bay of parabolic cross-section

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**Abstract.** Run-up of solitary waves of different bell-like shapes (solitary-like and Lorentz-like waves and sine-like pulses) is studied in a linearly inclined bay of parabolic cross-section. Their maximum run-up heights, maximum water flow velocities, and parameters of wave breaking on the beach are calculated, compared, and discussed. It is shown that these parameters for different pulses of the same height and characteristic wavelength coincide with an acceptable accuracy, hence allowing parameterization of the corresponding formulas for run-up characteristics.

**Key words:** nonlinear shallow water theory, long wave run-up on a beach, bay of parabolic cross-section.

## 1. INTRODUCTION

Reliable estimation of the tsunami run-up characteristics on a beach is the key problem of the tsunami warning, prevention, and mitigation, as it would allow defining the tsunami inundation zone and impact on port and coastal structures.

Theoretically, the most studied is wave run-up on a plane beach. First rigorous mathematical results for long wave run-up were obtained by Carrier and Greenspan (1958) for the beach of a constant slope. After their study, several exact analytical solutions to this problem for various shapes of incident waves have been found (Spielvogel, 1975; Pedersen and Gjevik, 1983; Synolakis, 1987; Pelinovsky and Mazova, 1992; Tadepalli and Synolakis, 1994; Brocchini and Gentile, 2001; Carrier et al., 2003; Kânoğlu, 2004; Tinti and Tonini, 2005; Kânoğlu and Synolakis, 2006; Antuono and Brocchini, 2007, 2008, 2010; Didenkulova et al., 2007b; Madsen and Fuhrman, 2008; Didenkulova, 2009). However, in practice the shape of the incoming tsunami wave is usually unknown and its parameters (wave height and wavelength) are either pre-computed using different hydrodynamic models or estimated from the first wave measurements, such as DART buoys. That is why it is extremely important to have formulas that could give fast and reliable estimates of the inundation zone and flow velocity on the beach based on these rough preliminary data of the incoming wave characteristics.

Following this idea, the influence of an incident wave shape on tsunami run-up characteristics (maximal run-up height, shoreline velocity, and breaking parameter) has been studied (Didenkulova et al., 2007a; Didenkulova and Pelinovsky, 2008). It was shown that symmetric bell-shaped pulses of slightly different shape, such as sinusoidal, solitary-like, and Lorentz-like pulses, produce similar tsunami run-up characteristics; so the influence on the wave shape can be parameterized.

However, estimates of run-up characteristics, calculated for a plane beach, are not always optimal as it was observed during the Samoa 2009 and Japan 2011 tsunamis, where the observed run-up height significantly exceeded the one estimated by the formulas for a

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plane beach (Okal et al., 2010). In both these cases the tsunami was propagating in a U-shaped bay. Later (Didenkulova, 2013) it was shown that estimates of run-up characteristics performed using the U-shaped bay approach are in a good agreement with observations of tsunami run-up height in Pago-Pago during the Samoa 2009 tsunami. Hence, it is necessary to consider U-shaped bays separately and develop the corresponding explicit estimates of tsunami run-up characteristics.

In this paper we study the run-up of bell-shaped waves on the coast of an inclined bay of parabolic cross-section in the framework of nonlinear shallow water theory. The paper is organized as follows. In Sections 2 and 3, following Didenkulova and Pelinovsky (2011a, 2011b) we briefly describe long wave propagation and run-up in a parabolic bay. Parameterization of major run-up characteristics is introduced and discussed in Section 4. Main results are summarized in the Conclusions.

## 2. LONG WAVE DYNAMICS IN A PARABOLIC BAY

Let us consider a bay of parabolic cross-section (Fig. 1).

The geometry of the bay, shown in Fig. 1, is described by the following formula

$$z(x, y) = -h(x) + \frac{y^2}{y_0} = -\alpha x + \frac{y^2}{y_0}, \tag{1}$$

where $h(x) \geq 0$ is the undisturbed water depth along the main channel axis, where $x$-axis is directed offshore, $\alpha$ is the beach slope along the main channel axis, and $y_0$ is the effective width of the channel.

We underline that this type of the bay is also often observed in the Nature in fjords and underwater canyons. Examples of such bays were shown by Didenkulova and Pelinovsky (2011b).

Nonlinear shallow water equations for a bay of parabolic cross-section are (Didenkulova and Pelinovsky, 2011a)

$$\frac{\partial H}{\partial t} + u\frac{\partial H}{\partial x} + \frac{2H}{3}\frac{\partial u}{\partial x} = 0, \quad \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial H}{\partial x} = g\alpha, \tag{2}$$

where $H(x, t) = \eta(x, t) + h(x)$ is the total depth along the main channel axis, $\eta(x, t)$ is water surface displacement, $u(x, t)$ is water flow averaged over channel cross-section, and $g$ is gravitational acceleration.

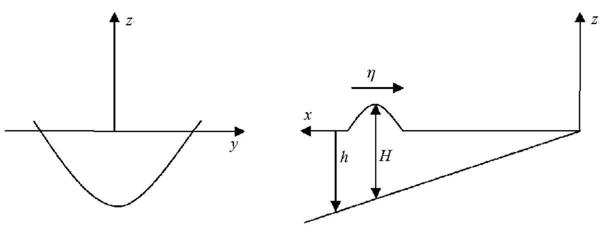


**Fig. 1.** Linearly inclined bay of parabolic cross-section.

Equations (2) differ from classical ID shallow water equations (Didenkulova, 2009) only by an additional coefficient 2/3 in the first equation, which is determined by the parabolic shape of the channel cross-section.

Using Riemann invariants for parabolic channel,

$$I_{\pm} = u \pm \sqrt{6gH} - gt\alpha, \tag{3}$$

and hodograph transformation, the system of nonlinear hyperbolic Eqs (2) can be reduced to a linear wave equation

$$\frac{\partial^2 \Phi}{\partial \lambda^2} - \frac{\partial^2 \Phi}{\partial \sigma^2} - \frac{2}{\sigma}\frac{\partial \Phi}{\partial \sigma} = 0. \tag{4}$$

Then all desired variables are expressed through the function $\Phi$,

$$u = \frac{1}{\sigma}\frac{\partial \Phi}{\partial \sigma}, \quad \eta = -\frac{1}{g}\left(\frac{u^2}{2} - \frac{1}{3}\frac{\partial \Phi}{\partial \lambda}\right), \tag{5}$$

$$x = \frac{1}{g\alpha}\left(\frac{u^2}{2} + \frac{\sigma^2}{6} - \frac{1}{3}\frac{\partial \Phi}{\partial \lambda}\right), \quad t = \frac{u - \lambda}{g\alpha}. \tag{6}$$

Detailed derivation of Eqs (4)–(6) is given in (Didenkulova and Pelinovsky, 2011a). New variables $\sigma$ and $\lambda$ represent generalized coordinates. From Eqs (5)–(6) it is possible to define the physical meaning of $\sigma$. It is always positive and directly related to the total water depth along the main channel axis $\sigma = \sqrt{6gH}$. Thus, Eq. (4) should be solved at semi-axis $\sigma \geq 0$. The physical meaning of $\lambda$ is not so transparent, but for the initial condition of zero-velocity the time moment $t = 0$ also corresponds to $\lambda = 0$.

We remind that Eqs (4)–(6), which describe wave dynamics in a parabolic channel, deal with variables averaged over channel cross-section. At the same time, the distribution of water surface in space can be found from Eq. (1):

$$y(x, t) = \pm\sqrt{y_0 H(x, t)}. \tag{7}$$

Natural boundary conditions for the description of wave run-up on a beach are boundedness of the wave field (water flow velocity and displacement) far offshore ($\sigma \to \infty$ or $H \to \infty$) and at the coast ($\sigma = 0$ or $H = 0$). Initial conditions for the function $\Phi$ depend on the initial conditions for water flow velocity and displacement. If at the initial time moment the velocity field is zero $u(x, t = 0) = 0$, the initial time $t = 0$ also corresponds to zero value of the variable $\lambda = 0$. Therefore, the initial conditions for the function $\Phi$ are

$$\Phi\big|_{\lambda=0} = 0, \tag{8}$$

$$\partial\Phi/\partial\lambda\big|_{\lambda=0} = -3g\eta[x(\sigma,0)] = \sigma^2/2 - 3g\alpha\, x(\sigma)\big|_{\lambda=0}, \quad (9)$$

where $x(\sigma)\big|_{\lambda=0}$ is the initial state of the water surface, which can be found by knowing the total water depth at the initial moment of time $H(x,0)$ and using $\sigma = \sqrt{6gH}$.

The solution of the wave equation for the corresponding initial and boundary conditions can be found in the following form:

$$\Phi(\sigma,\lambda) = \frac{[\Theta(\lambda+\sigma) - \Theta(\lambda-\sigma)]1(\lambda-\sigma) + [\Theta(\lambda+\sigma) - \Theta(\sigma-\lambda)]1(\sigma-\lambda)}{\sigma}, \quad (10)$$

where

$$\Theta(\zeta) = \frac{\zeta^4}{16} - \frac{3}{2} g\alpha \int_0^{\zeta} \sigma\, x(\sigma)\big|_{\lambda=0}\, d\sigma, \quad (11)$$

and $1(\ldots)$ is the Heaviside function.

Formulas (10)–(11) allow explicit description for nonlinear dynamics of the moving shoreline. The major difference of the described wave dynamics in a parabolic bay from the case of a plane beach (Carrier and Greenspan, 1958) is that in a linearly inclined bay of parabolic cross-section waves propagate without inner reflection from the sea bottom (travelling waves in the nonlinear problem) and reflection occurs only from the coast (shoreline). Therefore, all changes of wave dynamics, including transformation of the wave and its characteristics, also occur only in the immediate vicinity of the shoreline. This feature, characteristic for all non-reflecting geometries, leads to abnormal wave amplification at the coast (Didenkulova et al., 2006, 2008; Didenkulova and Pelinovsky, 2011b).

## 3. WAVE RUN-UP IN A PARABOLIC BAY

In the linear approach, wave run-up on a beach in a bay of parabolic cross-section simply depends on the time derivative from the initial water displacement $\eta_{\text{in}}$ (Didenkulova and Pelinovsky, 2011a),

$$R = 4\sqrt{\frac{3}{2}}\frac{L}{\sqrt{gh_0}}\frac{d\eta_{\text{in}}}{dt}, \quad (12)$$

where $L$ is the distance from the initial wave location to the coast, $h_0$ is water depth at the distance $L$ from the coast, and $g$ is gravity acceleration.

The horizontal velocity of the moving shoreline $U(t)$ can be found from

$$U(t) = \frac{1}{\alpha}\cdot\frac{dR}{dt}, \quad \alpha = \frac{h_0}{L}. \quad (13)$$

Note that even though formulas (12)–(13) describe wave height on a beach in the linear approximation, they are also basic for the calculation of run-up characteristics within a nonlinear framework (Didenkulova and Pelinovsky, 2011a). As it is demonstrated in (Didenkulova and Pelinovsky, 2011a), maxima of wave run-up heights and shoreline velocities in linear and nonlinear problems coincide. Since in this study we are interested only in maximal values of run-up characteristics, it is enough to find maxima of functions (12) and (13).

One of the major parameters characterizing long wave run-up on a beach is the wave breaking parameter $Br$. It is expressed through time derivative from the shoreline velocity found in linear approximation

$$Br = \frac{1}{g\alpha}\max\left[\frac{dU}{dt}\right], \quad (14)$$

and $Br < 1$ for non-breaking waves and $Br \ge 1$ for breaking waves.

## 4. PARAMETERIZATION FORMULAS FOR WAVE RUN-UP CHARACTERISTICS IN A PARABOLIC BAY

For our analysis we consider the same types of initial bell-shaped waves as in (Didenkulova and Pelinovsky, 2008), which are soliton-like waves

$$\eta_{\text{in}}(t) = A\,\mathrm{sech}^n(t/T), \quad n = 1, 2, \ldots, 20, \quad (15)$$

sinusoidal pulses

$$\eta_{\text{in}}(t) = A\cos^n\left(\frac{\pi t}{T}\right), \quad n = 3, 4, \ldots, 20, \quad (16)$$

and Lorentz-like waves

$$\eta_{\text{in}}(t) = \frac{A}{(1+(t/T)^2)^n}, \quad n = 1, 2, \ldots, 20. \quad (17)$$

To study the possibility of parameterization of wave run-up characteristics for impulses (15)–(17), we introduce numerical coefficients $\mu_R$, $\mu_{U\pm}$, and $\mu_{Br}$, which depend on the shape of each particular wave, so that Eqs (12)–(14) for extreme wave run-up characteristics can be rewritten in the following way:

$$R_{\max} = \mu_R R_0, \quad R_0 = 4\sqrt{\frac{3}{2}}\frac{AL}{\sqrt{gh_0}T_{\text{eff}}}, \quad (18)$$

$$U_{\max} = \mu_{U+}U_0, \quad U_{\min} = \mu_{U-}U_0, \quad U_0 = 4\sqrt{\frac{3}{2}}\frac{AL}{\alpha\sqrt{gh_0}T_{\text{eff}}^2}, \quad (19)$$

$$Br = \mu_{Br} Br_0, \quad Br_0 = 4\sqrt{\frac{3}{2}} \frac{AL}{g\alpha^2 \sqrt{gh_0} T_{\mathrm{eff}}^3}, \qquad (20)$$

where $T_{\mathrm{eff}}$ is the effective duration of the wave defined at the 2/3 level of the maximal initial wave height, which is similar to the definition of the significant wave in oceanography, $R_{\max}$ is maximal run-up height, and $U_{\max}$ and $U_{\min}$ are maximal run-up and back-wash velocities, respectively. Note that the maximal back-wash in a parabolic bay coincides with $R_{\max}$.

Thus, the influence of the initial wave shape on run-up characteristics is concentrated in the parameters $\mu_R$, $\mu_{U\pm}$, and $\mu_{Br}$. Calculations of these parameters for different sets of bell-shaped pulses (15)–(17) are shown in Figs 2–4.

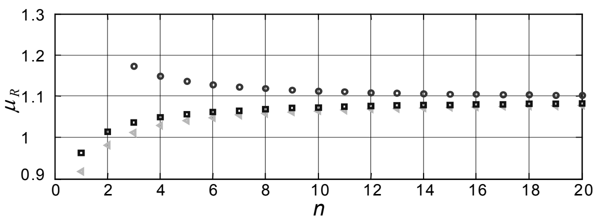


**Fig. 2.** Wave shape parameter for maximal run-up height: circles correspond to initial pulses of sinusoidal shape, grey triangles to Lorentz-like waves, and squares to soliton-like waves.

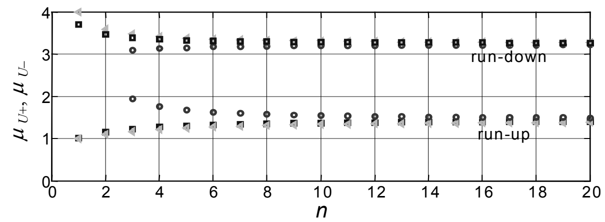


**Fig. 3.** Wave shape parameter for maximal run-up and run-down velocities: circles correspond to initial pulses of sinusoidal shape, grey triangles to Lorentz-like waves, and squares to soliton-like waves.

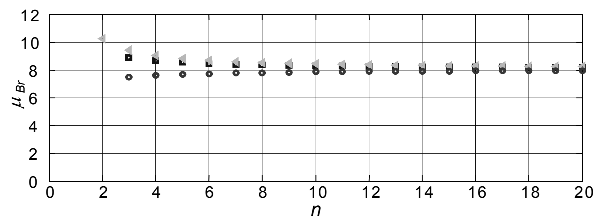


**Fig. 4.** Wave shape parameter for the breaking parameter *Br*: circles correspond to initial pulses of sinusoidal shape, grey triangles to Lorentz-like waves, and squares to soliton-like waves.

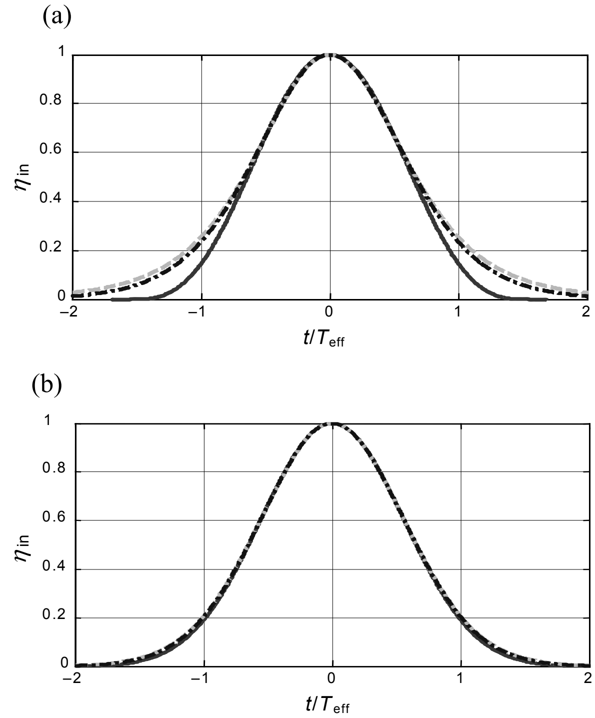


**Fig. 5.** Shapes of the initial waves approaching the coast for $n = 3$ (a) and $n = 20$ (b): solid lines correspond to impulses of sinusoidal shape, grey dashed lines to Lorentz-like waves, and dash-dotted lines to soliton-like waves.

It follows from Fig. 2 that with an increase in $n$, the parameter $\mu_R$ for pulses of all considered types tends to the same value $\approx 1.1$ with maximum spread of values below 15%. At the same time, maximal differences in $\mu_R$ are observed for small $n$, when differences in wave shape are maximal. This is well demonstrated in Fig. 5, where shapes of the initial waves approaching the coast for $n = 3$ and $n = 20$ are illustrated. It can be seen that all considered wave types tend to a certain unified wave shape with an increase in $n$.

Parameters $\mu_{U\pm}$ for maximal run-up and run-down velocities also tend to the same values for all three sets of pulses, which are equal to $\approx 1.4$ and $\approx 3.2$, respectively (Fig. 3). It can also be seen from Fig. 3 that the run-down velocity is more than twice higher and at the same time more stable than the run-up velocity. So, the spread of the values of the parameter $\mu_{U-}$ for run-down velocity does not exceed 12%, while the corresponding spread of the values for run-up velocity is 56%. This means that even small changes in the shape of the wave approaching the beach may significantly influence the run-up velocity.

Finally, as it has been expected, $\mu_{Br}$ for the breaking parameter for large $n$ also tends to the unique constant value $\approx 8.1$ with the maximum spread of the values of 24% (Fig. 4). As it is known, the first wave breaking always occurs at the run-down stage (Pelinovsky, 1982). Hence, it is expected that $\mu_{Br}$ for the breaking parameter behaves similarly to the parameter $\mu_{U-}$ for the maximal run-down velocity.

## 5. CONCLUSIONS

Run-up of different bell-shaped pulses propagating in a linearly inclined bay of parabolic cross-section has been studied analytically in the framework of shallow water theory. It is shown that characteristics of the tsunami run-up on the coast (maximal run-up height, shoreline velocity, and breaking parameter) are within the same limits for all considered bell-shaped waves and can be parameterized. For example, the variation in the maximal tsunami run-up height and back-wash velocity does not exceed 15% and 12%, respectively. Therefore, definition of wave length (wave duration) at the 2/3 level of maximum wave height (similar to the definition of significant wave in oceanography) is optimal for the parameterization of run-up formulas in both a plane beach and a parabolic bay.

The final parameterized formulas for tsunami run-up characteristics in U-shaped bays with values for $\mu$, taken from Figs 2–4, are

$$R_{\max} = 5.4\frac{AL}{\sqrt{gh_0}T_{\text{eff}}}, \quad U_{\max} = 6.8\frac{AL}{\alpha\sqrt{gh_0}T_{\text{eff}}^2},$$
$$U_{\min} = 15.7\frac{AL}{\alpha\sqrt{gh_0}T_{\text{eff}}^2}, \quad Br = 39.7\frac{AL}{g\alpha^2\sqrt{gh_0}T_{\text{eff}}^3}. \tag{21}$$

For comparison we reproduce here the corresponding parameterized formulas for a plane beach (Didenkulova and Pelinovsky, 2008):

$$R_{\max} = 3.5A\sqrt{\frac{L}{\sqrt{gh_0}T_{\text{eff}}}},$$
$$R_{\min} = 1.5A\sqrt{\frac{L}{\sqrt{gh_0}T_{\text{eff}}}},$$
$$Br = 13\frac{H_0L}{\alpha gh_0T_{\text{eff}}^2}\sqrt{\frac{L}{\sqrt{gh_0}T_{\text{eff}}}}, \tag{22}$$
$$U_{\max} = 4.5\frac{AL}{T_{\text{eff}}}\sqrt{\frac{1}{\alpha h_0\sqrt{gh_0}T_{\text{eff}}}},$$
$$U_{\min} = 7\frac{AL}{T_{\text{eff}}}\sqrt{\frac{1}{\alpha h_0\sqrt{gh_0}T_{\text{eff}}}}.$$

Since the general solution for wave run-up on a plane beach differs from the one in a U-shaped bay and is expressed through Bessel functions (see, Didenkulova, 2009 for details), the parameterization in Eqs (22) is also different.

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## Kellakujuliste madala vee üksiklainete lainerünnaku omadused paraboolse ristlõikega lahtedes

Oleg Didenkulov, Ira Didenkulova ja Efim Pelinovsky

On analüüsitud klassikalise kellakujulise profiiliga üksiklainete lainerünnaku omadusi kaldpõhja ja paraboolse ristlõikega lahes. Vaatluse all on Kortewegi-de Vriesi solitoni, siinusekujulise profiiliga ja algebralise profiiliga (nn Lorentzi tüüpi) üksiklainete üldistused. On arvutatud lainerünnaku maksimaalne kõrgus, vee liikumise maksimaalne kiirus ja laine murdumist iseloomustavad suurused kõnesolevate profiilide naturaalarvuliste astmete jaoks. On näidatud, et need suurused on praktiliselt võrdsed ja ühesuguse algkõrguse ning ruumilise ulatusega, kuid erineva kujuga solitonide jaoks, ja on tuletatud solitonide kujust sõltumatud seosed lainerünnaku mitmesuguste parameetrite jaoks.