



# Currency Hedge – Walking on the Edge?

Fabio Filipozzi, Kersti Harkmann

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# Currency Hedge – Walking on the Edge?

Fabio Filippozi and Kersti Harkmann\*

## Abstract

We study whether it is possible to find optimal hedge ratios for a foreign currency bond portfolio to lower significantly the risk and increase the risk adjusted return of a portfolio. The analysis is conducted from the perspective of euro area based investors to whom short-selling restrictions might apply. The ordinary least squares approach is challenged with the optimal hedge ratios found by the DCC-GARCH approach in order to investigate whether time-varying hedging is superior to the standard constant hedge ratios found by OLS. We find that hedging significantly lowers the portfolio risk in domestic currency terms and improves the Sharpe ratios for both single instrument and equally weighted multi asset portfolios. Optimal hedging using the standard OLS approach and using time-varying hedging give similar results, the latter being superior to the first in terms of risk-adjusted return.

JEL Codes: C32, C58, G11, G15, G23, G32

Keywords: optimal hedge ratios, portfolio risk hedging

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The views expressed are those of the authors and do not necessarily represent the official views of Eesti Pank.

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## **Non-technical summary**

The official foreign exchange reserves holdings have been growing for years and are around five times as large as they were in the 1980s. These reserves are divided between different currencies and invested largely in government bonds. Managing the official foreign exchange reserves involves many implications. The official institutions need to compromise between different goals of the official foreign exchange reserves and investment policies and limits. A correct balance must be kept between the earning, liquidity and risks. Having invested the reserves in foreign bonds brings risk which must be dealt with as it is generally known that while the exchange rate returns are lower than the returns of the bond indices in the local currencies, their volatility is significantly higher than the volatility of the bonds. This extra volatility could be lowered by hedging the foreign currency exposure.

Since the issue of how much currency exposure should be hedged was initially addressed, the literature on exchange rate risk hedging has developed along various paths. The early literature mainly uses simple conventional methods to find the optimal hedge ratio, applying linear regressions to minimize the variance or using naïve hedging strategies in which the hedge ratio is chosen without any optimization. Nevertheless, the studies have given contradictory results on whether there is an optimal hedge ratio or how to find it.

The current paper analyses whether and to what extent foreign bond investments should be hedged in order to minimize the variance of the overall portfolio. We have taken the perspective of the euro area investor and have used a weekly frequency. Starting with a comparison of unhedged and fully hedged portfolio risk-return profiles, we then proceed to find constant optimal hedge ratios using the conventional minimum variance framework, under which the investor is risk adverse and wishes to minimize the volatility of the portfolio. Due to the time varying nature of the asset returns, the analysis is expanded with the use of the multivariate time series DCC-GARCH approach. We have also taken account of the possible short selling restrictions which could be applied and have analyzed the questions under observation for both single bond and multiple bond portfolios.

We show that portfolios of foreign bond investments are sensitive to foreign currency risk and these risks can only partially be mitigated in multiple asset portfolios. In line with the previous literature on international fixed income portfolios, we found that any kind of hedging improves the risk-return profiles in our sample significantly and the constant hedge ratio tends to dominate slightly. Unhedged positions perform the worst in terms of the volatility of the domestic-currency value of the portfolios.

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# 1. Introduction

According to the IMF, the official foreign exchange reserves holdings reported by 146 countries and entities in the fourth quarter of 2013 totalled 11,673,628 million US dollars. These reserves have been growing for years and are around five times as large as they were in the 1980s (Morahan and Mulder (2013)). These reserves are divided between different currencies but the majority is held in the top seven most popular ones and invested largely in government bonds (Morahan and Mulder (2013)). The daily trading turnover of the foreign exchange market has grown steadily over the years and averaged 5.3 trillion US dollars in April 2013 (Bank for International Settlements (2013)).

The literature has not reached a conclusion about how much of the foreign currency exposure should be hedged, nor about which estimation method should be used to estimate the hedge ratios. It is typically shown, however, that hedging currency exposure reduces the volatility of bond portfolios and sometimes even adds extra return and might improve the risk-return trade-off. The evidence is not so clear-cut for stocks. The question of whether the foreign exposure should be fully hedged or whether it is possible to optimize the level of hedge has not been clearly answered and remains a topic of debate.

Our paper contributes to the existing literature in two ways. First, we conduct the analysis of a multicurrency portfolio, with several different government bond indices and currencies, while existing literature focuses mainly on two or three currencies. Considering a wider portfolio makes the analysis closer to reality and also allows account to be taken of the effects of cross correlations between the different currencies and bonds. Second, we make our analysis closer to real applicable situations by imposing no short selling of currencies, often a constraint for investors, and supposing a weekly hedging of the currency exposure (whereas a daily hedging strategy is often supposed in the literature). Our analysis will show that, with assumptions closer to reality, both no hedge and full hedge are sub-optimal strategies, and investors can achieve better risk adjusted return by optimizing (statically or dynamically) their currency exposure.

We address the question of hedging policy for official institutions, taking the view of a euro base investor. Given that official reserves are mostly invested in fixed income securities, we will focus on a bond portfolio invested in non-euro markets. The currencies chosen are those which constitute the typical official institution portfolio. These are USD, JPY, GBP, AUD, CAD, NOK and CHF. We will address at first the degree of hedging needed to minimize the variance of return of the portfolios and secondly we will com-

pare the results arrived at by different methods for calculating the hedging ratios. We contribute to the existing literature on the question of optimal hedging in the following ways. We argue that the hedge ratios found for single currencies do not give correct guidance for what the hedge ratios of a multi asset portfolio should be. We develop the analysis further by employing time-varying hedge ratios found by the DCC-GARCH method, which has been used in the literature but only on portfolios consisting of one or two currencies. The results indicate that hedging does reduce the risk but the degree of reduction depends on the method employed. While the literature often suggests a 100% hedge for bond portfolios, our results shows that hedge ratios found with OLS and DCC-GARCH both decrease the variance of the portfolio and improve the risk adjusted return. Furthermore, we use weekly data as it is simply not feasible to use daily data for hedging. It is clear that the level of the appropriate hedging strategy also depends on the profile of the investor and whether considerations of risk or return dominate.

The remaining part of the article is organized as follows. The next section provides a description of the data and the methodology. Then we present the empirical results of the analyses. The last section summarizes with discussion and conclusions.

## **2. Literature review**

Since the issue of how much currency exposure should be hedged was initially addressed (among others by Perold and Schulman (1988) and Black (1989)), the literature on exchange rate risk hedging has developed along various paths. The early literature mainly uses simple conventional methods to find the optimal hedge ratio, applying linear regressions to minimize the variance or using naïve hedging strategies in which the hedge ratio is chosen without any optimization. Nevertheless, the studies have given contradictory results on whether there is an optimal hedge ratio or how to find it. Perold and Schulman (1988) discussed the importance of the foreign currency exposure for the performance of portfolios containing both stocks and bonds and showed that during volatile periods the risks associated with currencies should not be overlooked. They argue that in order to deal with the risks stemming from volatile currencies, and assuming that the currency returns are zero in the long run, it would be optimal to hedge the exposure. They conclude that hedging 100% of the currency exposure reduces the volatility of portfolio return without having a negative impact on the expected return of the portfolio. Black (1989) came to the conclusion that under certain assumptions, all investors should have the same hedge ratio irrespective of their portfolio choice, and he derived a universal formula for hedging.

In the more recent literature, these conclusions have been challenged. It was subsequently shown that the 100% hedge ratio might not be optimal for reducing the volatility of the portfolio returns (mainly thanks to the cross correlations between different currencies) and that the optimal hedge ratios differ depending on the asset classes and currency pairs, meaning there is no universal hedge ratio. Glen and Jorion (1993) argued that hedging might not improve the return of a portfolio consisting of stocks and bonds and that the final results of hedging depend on the restrictions imposed and the strategy chosen. However, hedged portfolios tend to show lower levels of volatility and conditional hedging outperforms the fully hedged portfolios. They also argue that hedging without constraints may lead to extreme hedge ratios for both short and long positions. This implies that the hedging optimisation should be made under the restrictions which matter the most, such as whether investors are allowed to speculate with currency or whether the hedge ratio must be equal to or below the foreign currency exposure in the portfolio.

Haefliger, Wälchli and Wydler (2002) argue that the bond portfolio should be hedged 100% in order to reduce the portfolio risk stemming from the currency exposure while the stock portfolio hedge could be less or even zero, and in the same vein, De Roon, Eiling, Gerard and Hillion (2012) show that hedging decreases the portfolio volatility significantly. However, hedging not only influences the variance but might also change the expected return, and so the authors use the Sharpe ratios to evaluate the impact of hedging on the risk-return trade-off. Hedging also seems to worsen the portfolio skewness and kurtosis and so hedging does not lower the volatility without a price; risk can be reduced for the loss of return.

There has been discussion whether the investment horizon of the investor is important for a hedging decision. Froot (1993) concludes that hedging only makes sense with a short horizon but not with long horizons because hedging with a horizon of five years or more tends to increase the variance of the portfolios for stocks and bonds. This result stems mainly from the fact that factors driving the exchange rate changes in the short terms are different from the factors driving changes in the long term. Campbell, Viceira and White (2003) conclude that short-term bond investors should always hedge contrary to long-term investors. Carcano (2007) shows that in the short term, hedging outperforms no hedging and lowers the volatility of the portfolio but in the longer term, hedging is not optimal for a foreign bond portfolio that also contains stocks and domestic bonds. He also emphasizes that the hedge costs matter.

Campbell, Medeiros and Viceira (2010) conclude that bond investors should always hedge 100% in order to lower volatility and that it is possible to find optimal hedge ratios. These results hold for both the short-term and longer-term investment horizons. The hedging decision for an equity portfo-



lio should be made considering the correlations between currencies and equities as a currency investment might be even more beneficial than hedging.

Schmittmann (2010) studies the impact of currency hedging on the stock and bond portfolios from different country and time horizon perspectives and also for single and multi-asset portfolios. He concludes that bond portfolios should be fully hedged for the purpose of volatility reduction for both the short and the long horizon and that the returns do not differ significantly. This holds even if the investment horizon is lengthened to over five years. For the equity portfolios, the results are not so clear and the optimal hedge ratio for stocks depends on the currency used.

It has been discussed though that the distribution characteristics of the asset returns series mean the conventional OLS method might be too restrictive and other methods which take the time-varying nature of the returns into account should be studied. As alternatives to the OLS approach, multivariate conditional variance models and other methods have been used. Kroner and Sultan (1993) include the error-correction model with the GARCH model and study the hedge ratios with futures only for the spot exchange rates of the British pound, Canadian dollar, Deutsche mark, Japanese yen and Swiss franc from 1985 to 1990. They find that their daily dynamic hedging approach offers better results than the conventional methods in terms of risk. Ku, Chen and Chen (2007) use the ordinary least square, error correction, constant conditional correlation, and dynamic conditional correlation (DCC) models to study daily hedging with futures in the British and Japanese currency markets. They conclude that hedging effectiveness with DCC is superior to that with the other methods for the period studied between 1998 and 2004. Lien, Tse and Tsui (2002) parallel the daily hedging performance estimated with the OLS method and with the constant-correlation VGARCH model and conclude that for the single asset portfolio, the OLS outperforms the alternative time-varying approach for currency, commodity and stock markets, meaning the variance of the portfolio hedged with the OLS hedge ratio is smaller than that of the dynamic hedge ratio portfolio, though the difference can be considered small. However, these results are based on very simple portfolios consisting only of single assets and no bonds or equities are included, nor are portfolios with several assets analyzed.

These limitations of the literature are addressed in the more recent literature. Caporin, Jimenez-Martin and Gonzalez-Serrano (2013), analyze the optimal hedge ratios for portfolios consisting of two foreign bonds or equities by using the DCC and several other multivariate GARCH models. The authors take the perspective of a euro area investor who invests in British and US assets but still has a home bias. The hedging is done with futures and daily frequency is used. The results are not conclusive as the portfolio of two assets can be considered too restrictive to replicate fully the issues of real

world portfolios. Nevertheless the results indicate that using the time-varying hedge ratio can improve the hedging effectiveness and the other performance measures but the result is not conclusive.

Opie, Brown and Dark (2012) study the hedging of portfolios consisting of multiple stock market returns with several methods from the US investor's perspective. The daily DCC model is used for the period 2002 to 2010 to find time-varying hedge ratios and it is shown that using these ratios to minimize the portfolio risk outperforms the hedging done with the conventional OLS method but the results depend on the portfolio and currency.

Lien (2009) argues that the hedging effectiveness of different methods depends on the sample size. He shows that the hedge ratios found by the ordinary least square method perform better in a large sample as the unconditional variance is minimized. For smaller samples, the variations in the variance of portfolio returns might demand the use of time-varying hedge ratios and thus the time-varying hedging outperforms the result of the conventional OLS method.

While there is a strong indication that hedging either optimally or fully reduces the volatility of bond portfolios, there is no clear answer as to what the optimal level of hedge should be for bond portfolios, nor to the question of how to determine the optimal level of hedge. The majority of the earlier studies analyze the issue by looking at single currency pairs without addressing the construction of portfolios. Neither is dynamic hedging of the bond portfolios addressed in detail, as studies such as Choudhry (2004), Ku, Chen and Chen (2007) and Lien (2009) focus on stocks and foreign currency and on the use of futures as hedging instruments. Furthermore, the daily hedging strategies can be considered unfeasible in practice. In the vast part of the literature, the hedging is done by using futures and very few investigations are available where forward contracts have been used, which could be considered a limitation<sup>1</sup>. A method built on the Engle (2002) dynamic conditional correlation method has found some support but in general there is not enough evidence to conclude that the use of dynamic hedging helps to improve the hedge effectiveness.

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<sup>1</sup> Forward contracts might prove to be more flexible in practice as futures are in fact highly standardized for size and expiration.

### 3. Methodology

This section presents the dataset and the methodology. Let  $S_t$  be the spot exchange rate of the base currency per unit of foreign currency<sup>2</sup> and  $P_t$  be the index value of the foreign bond index at time  $t$ . In this case  $e_t$  is the change in the exchange rate between the domestic and foreign currencies  $e_t = \frac{S_t}{S_{t-1}} - 1$  and the return of the foreign currency bond index investment in local currency is  $x_t = \frac{P_t}{P_{t-1}} - 1$ .

The total return and risk of the foreign denominated investment is the product of the two elements: the return of the asset (in foreign currency) and the profit or loss due to the change in the exchange rate between the domestic and the foreign currencies, and their cross product. Thus the unhedged return of a single asset portfolio of one unit of foreign investment can be written as:

$$r_{u,t} = (1 + x_t)(1 + e_t) - 1 = x_t + e_t + x_t e_t \quad (1)$$

The currency risk can be hedged using forward contracts, so at  $t - 1$  it is possible to sell the foreign currency for period  $t$  at a fixed rate  $F_{t-1,t}$ . The return for period  $t$  from the hedging position is  $f_t = \frac{F_{t-1,t}}{S_{t-1}} - 1$ . The total return of the foreign investment where  $h_{t-1}$  is the hedge ratio can be written as  $r_{h,t} = r_{u,t} + h_{t-1}(f_t - e_t)$ . The second term of the total hedged return represents the difference between the return from the hedging and the exchange rate return. This will usually be different from zero because forward exchange rates are different from the realized spot exchange rates. We will call this difference the net return from hedging.

If the investor has hedged 100%, the total return of the fully hedged portfolio can be written as  $r_{h,t} = x_t + e_t x_t + f_t$  i.e. the unhedged return of the investor is adjusted for the profit or loss due to the exchange rate change and the forward premium or discount<sup>3</sup>, or the net return from hedging.<sup>4</sup> It is worth noting here that the hedging through forwards implies a cost for the investor, mainly coming from the bid offer spread paid every time the hedge is rolled over. We consider the cost stemming from hedging activity to be relatively small, and do not consider it in detail throughout our analysis.

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<sup>2</sup>  $S_t$  is the amount of domestic currency needed to buy 1 unit of the foreign currency (i.e.  $e_t$  will be positive when the foreign currency appreciates against the local currency, adding to the performance of the financial asset).

<sup>3</sup> We follow the literature and hedge the initial investment amount, and not the whole exchange rate exposure.

<sup>4</sup> We have used simple returns, not logarithmic returns.

In the traditional portfolio optimization framework, investors maximize the return of their portfolio by changing the weights of the possible asset classes, under the constraint of a maximum amount of risk/volatility they are ready to accept. Partly following the literature presented above, we deviate from this approach in many ways. First of all, we focus only on one part of the portfolio choice. We consider an investor with institutional constraints, which is “forced” to invest a given amount in fixed income securities. The second deviation from the traditional mean variance optimization framework is that our investor minimizes the variance of the portfolio instead of maximizing the risk adjusted return. This is typical in the literature of optimal hedging, and comes from the consideration that in the long term, currency exposure adds volatility to the portfolio without adding return. This means that currency exposure is controlled in order to avoid this “excess” volatility. The objective function minimized by our investor would be therefore the following:

$$\min_h \text{Var}(r_h) = \min_h \text{Var}(r_u - h_t[f_t - e_t]) \quad (2)$$

This is in line with the analysis made by Schmittmann (2010). The optimal hedge is given by the slope coefficient of the following regression, which is also used for the OLS estimations:

$$r_{u,t} = \alpha + \beta(e_t - f_t) + \varepsilon_t \quad (3)$$

where the coefficient  $\beta$  is the optimal hedge ratio estimated as  $\beta = \frac{\text{Cov}(r_{u,t}, e_t - f_t)}{\text{Var}(e_t - f_t)}$ . This set-up can easily be expanded to portfolios consisting of more than one asset.

We have taken the viewpoint of the euro area investor who has to hold an international bond portfolio. In our analysis we have used an equally weighted portfolio of seven assets and these weights are taken as given and no portfolio optimization is done. The reason for not relying on the 100% hedge ratio lies in the covariance-variance matrix. The hedge ratio in a portfolio of a single foreign currency asset depends only on the covariance of the foreign currency assets’ unhedged return with the currency return. The 100% hedge is optimal only in the special case where two variables are “identical”, or the return of the unhedged portfolio depends only on  $(e_t - f_t)$ . In all other cases the hedge ratio will be different from 100%. Furthermore, in the multi asset portfolio, the covariances with other assets’ and hedge returns’ must also be accounted for.

In the time-invariant framework, it is assumed that these covariance/variance matrices are constant over time and thus only one optimal hedge ratio exists. However, it is well documented in the literature that changes in vari-

ance of the asset returns might lead to incorrect conclusions about the constant hedge ratios as the regular OLS approach ignores the time-varying nature of the variance-covariance structure of the financial time series.

This concern is addressed in this paper using Engle's (2002) dynamic conditional correlation (DCC) approach, which takes changes in volatility into account by fitting GARCH models onto the individual variables. These adjustments might improve the quality of the optimal hedge ratio due to its dynamic nature, i.e. hedging ratios are no longer constant but change over time as the covariance-variance matrix is also time-variant.

For the sake of clarity, the DCC-GARCH method is presented in a bivariate case and the estimation results are obtained with the following steps, following the manner proposed in Engle (2002). First, the mean equations for the unhedged return  $r_{u,t}$  at time  $t$  are estimated by:

$$r_{u,t} = \mu_{r_{u,t}} + \varepsilon_{r_{u,t}} \quad (4)$$

where  $\mu_{r_{u,t}}$  is the constant term and  $\varepsilon_{r_{u,t}}$  is the residual term. The mean equations<sup>5</sup> for the net hedge cost ( $e_t - f_t$ ) at time  $t$  are estimated by:

$$(e_t - f_t) = \mu_{(e_t - f_t),t} + \varepsilon_{(e_t - f_t),t} \quad (5)$$

where  $\mu_{(e_t - f_t),t}$  is the constant term and  $\varepsilon_{(e_t - f_t),t}$  is the residual term. Then the conditional variances are estimated by GARCH(1,1) models:

$$h_{r_{u,t}} = \omega_{r_u} + \delta_{r_u} \varepsilon_{r_{u,t-1}}^2 + \gamma_{r_u} h_{r_{u,t-1}} \quad (5)$$

and

$$h_{(e_t - f_t),t} = \omega_{(e_t - f_t)} + \delta_{(e_t - f_t)} \varepsilon_{(e_t - f_t),t-1}^2 + \gamma_{(e_t - f_t)} h_{(e_t - f_t),t-1} \quad (6)$$

where  $\omega_i$  is the constant,  $\delta_i$  is the ARCH effect and  $\gamma_i$  is the GARCH effect. A positive parameter of  $\gamma_i$  shows clustering and persistence of volatility. Third, the dynamic covariance is found by:

$$h_{r_u(e_t - f_t),t} = \rho_{r_u(e_t - f_t),t} \sqrt{h_{r_{u,t}}} \sqrt{h_{(e_t - f_t),t}} \quad (7)$$

The model parameters are found with the maximum likelihood method and the dynamic optimal hedge ratio can be found by:

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<sup>5</sup> The final mean equations might be different for individual returns and currencies.

$$\beta_t = h_{r_u(e_t-f_t),t}/h_{(e_t-f_t),t} \quad (8)$$

A way to assess the effectiveness of hedging done by using the OLS and DCC-GARCH method hedge ratios is to compare the Sharpe ratios of the hedged portfolio with an alternative portfolio. This can be done using the  $z$ -statistic approach used in Kim (2012), developed by Jobson and Korkie (1981), and then adjusted by Memmel (2003), in which we calculate the Sharpe ratios for the portfolios and test whether the difference is statistically significant from the null hypothesis of no difference between the ratios. The  $z$  statistic can be found by

$$z = \frac{Sharpe_{r_h} - Sharpe_{r_u}}{\sqrt{\frac{1}{N} \left[ 2 - 2\tilde{\rho} + \frac{1}{2} (Sharpe_{r_u}^2 + Sharpe_{r_h}^2 - 2Sharpe_{r_u}Sharpe_{r_h}\tilde{\rho}^2) \right]}} \quad (9)$$

The Sharpe ratios can be calculated as  $Sharpe_{r_h} = \frac{\tilde{\mu}_{r_h} - r_f}{\tilde{\sigma}_{r_h}}$  and  $Sharpe_{r_u} = \frac{\tilde{\mu}_{r_u} - r_f}{\tilde{\sigma}_{r_u}}$ , where  $\tilde{\mu}_{r_h}$  and  $\tilde{\mu}_{r_u}$  are the mean returns of the hedged and unhedged portfolios,  $r_f$  is the risk-free interest rate which is assumed to equal zero in our analysis and  $\tilde{\sigma}_{r_h}$  and  $\tilde{\sigma}_{r_u}$  are the standard deviations of the return of the hedged and unhedged portfolios respectively,  $N$  is the number of observations used and  $\tilde{\rho}$  is the correlation between the return of the hedged and unhedged portfolios. The test statistic is asymptotically normally distributed<sup>6</sup>. The next section covers the results of the empirical analysis.

## 4. Data and descriptive statistics

The following section gives an overview of the data and the main descriptive statistics. The data are gathered with weekly frequency from Bloomberg (generic quotation) end of day mid-price. The data cover the period between 8 January 1999 and 27 December 2013. The last price in the market is used and the last available price is used for values that are missing due to holidays.

For the fixed income investment, we use the indices produced by Citigroup. Specifically, we take the total return index level with weekly frequency for each country with the Friday closing price for the World Government Bond Index for the 3 to 5 years maturity sector, measured in local currency. We gathered data for the following countries: the United States

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<sup>6</sup> See Jobson and Korkie (1981) and Memmel (2003).

(USD), Australia (AUD), Canada (CAD)<sup>7</sup>, Norway (NOK), Switzerland (CHF), Great Britain (GBP) and Japan (JPY). According to the BIS Triennial Central Bank Survey (Bank for International Settlements, 2013) these currencies are the top seven global currencies by daily market turnover. On top of that, these currencies also account for roughly 72% of the allocated global foreign exchange reserves according to the IMF (2014).

Table 1: Bond and exchange rate returns

<b><i>Bond Return in Local Currency</i></b>							
	AUD	CAD	CHF	UK	JPY	NOK	USA
Mean	0.109%	0.092%	0.053%	0.092%	0.025%	0.096%	0.092%
Std. dev.	0.477%	0.408%	0.284%	0.389%	0.195%	0.435%	0.479%
Risk/return	0.229	0.224	0.185	0.236	0.129	0.220	0.192
<b><i>Exchange Rate Return</i></b>							
	AUD	CAD	CHF	UK	JPY	NOK	USA
Mean	0.050%	0.033%	0.039%	-0.015%	0.001%	0.007%	-0.012%
Std. dev.	1.511%	1.425%	0.815%	1.166%	1.789%	0.966%	1.430%
Risk/return	0.033	0.023	0.048	-0.013	0.000	0.007	-0.008

Source: Bloomberg; authors' calculations.

Table 1 above reports the main statistics of the series employed in our analysis. It is worth noting that while the exchange rate returns are lower than the returns of the bond indices in the local currencies, their volatility is significantly higher than the volatility of the bonds. It follows that the volatility of the exchange rates might add risk to the portfolio of bonds denominated in foreign currencies, adding at the same time a relatively low return. The currency return from the UK and the USA from the perspective of the euro area investor is in fact negative and investors who are risk adverse could be especially sensitive to this kind of short-term volatility.

During the life of the single currency union, the euro itself has gone through several episodes when it has both appreciated and depreciated against other major currencies. There was a period during which the euro traded at parity with the US dollar in the first years after the paper currency was introduced. During the global financial crisis the euro depreciated against most of the currencies analyzed here, mostly because of the debt crisis that hit some of the euro area countries in 2010.

<sup>7</sup> For the Canadian dollar, the hedge cost is found as in Schmittmann (2010) assuming that the covered interest rate parity holds  $f_t = \frac{1+i_{d,t-1}}{1+i_{f,t-1}} - 1$ , where  $f$  is the forward premium and the  $i_{d,t-1}$  and  $i_{f,t-1}$  are the domestic and foreign currency interest rates respectively. For the missing observations between June and December 2013 the Bloomberg data are used.

## 5. Results

This section presents the results of the empirical analysis. Table 2 reports the hedge ratios found where first the optimal hedge ratios for the single instrument analysis are shown. The hedge ratios estimated by the OLS are somewhat higher than the average optimal dynamic hedge ratios for all the instruments, and are highest for the Swiss franc and lowest for the Australian dollar. We also see that the optimal hedge ratios for all the currencies remain below 100% except for the ratio for the Swiss franc.

Because short-selling restrictions might apply for investors, a hedge ratio above 100%, like that proposed for the Swiss franc, might not be acceptable and so we carry out the analysis with restrictions. For the standard OLS approach we have conducted a constrained optimization<sup>8</sup>. The results for the single instrument portfolio vary only for the Swiss franc and in that case, the optimal hedge ratio for the Swiss franc would be 100%.

Table 2: Optimal hedge ratios

<i>By single instrument</i>							
	AUD	CAD	CHF	UK	JPY	NOK	USA
Constant hedge	88.65%	92.21%	106.59%	94.32%	99.85%	93.69%	94.98%
Constant hedge constrained	88.65%	92.21%	100.00%	94.32%	99.85%	93.69%	94.98%
Average dynamic hedge	87.74%	89.88%	106.04%	93.10%	98.72%	93.64%	91.01%
Average dynamic hedge constrained	87.75%	89.86%	97.34%	91.85%	97.80%	92.99%	90.37%
<i>Portfolio</i>							
	AUD	CAD	CHF	UK	JPY	NOK	USA
Constant hedge	71.97%	93.06%	126.80%	107.09%	136.39%	96.21%	72.18%
Constant hedge constrained	68.72%	92.62%	100.00%	100.00%	100.00%	99.65%	100.00%
Average dynamic hedge	69.94%	87.91%	151.81%	109.08%	135.30%	94.04%	74.52%
Average dynamic hedge constrained	69.82%	85.27%	96.32%	95.33%	98.52%	89.16%	73.41%

Source: Bloomberg; authors' calculations.

Even though the average dynamic hedge ratios remain below 100%, the comparison of the constant and time-varying hedge ratios for single instruments is better illustrated in Figure 1, see below. As can be seen, there are

<sup>8</sup> We have estimated the model with the restriction so that the estimated hedge ratio coefficients should be between zero and 1.



periods where the dynamic hedge ratios move significantly above 100%. We have also found the dynamic hedge ratio by using a constraint so that the hedge ratio equals 100% every time the hedge ratio estimated by the model goes over 100%. The result of the average constrained dynamic hedge ratio is also shown in the table.

For the single instruments, the hedge ratios depend only on the variance-covariance of two variables but for the portfolio the variance-covariances between all the currencies and returns matter, which complicates the relationships. We see from the table that the hedge ratios for each currency change in the portfolio. The optimal hedge ratios found by OLS increase considerably for the Swiss franc and the hedge ratios also move over 100% for the British pound and the Japanese yen. If the analysis is conducted with the constraint of short-selling in addition to the constraints for currencies, the US dollar should also be hedged 100%. It is clear that the results of the single instrument analysis are not sufficient and these ratios cannot be directly applied if portfolio hedging is used.

As with the single instrument analysis, the average dynamic optimal hedge ratios for the portfolio optimization are very close to the constant hedge ratios, and slightly higher for the Swiss franc, the British pound and the US dollar. Nevertheless, as can be seen from Figure 2, the time-varying hedge ratios move to a larger extent than those for single instruments, especially for the Swiss franc and the Japanese yen. It is notable that the optimal dynamic hedge ratio for the US dollar also moves into negative territory at  $-23.14\%$ . In addition we have found the time-varying hedge ratios to be constrained similarly to those in the single instrument analysis.

Using these hedge ratios, we compare the results of various portfolio returns by single instrument and by equally-weighted portfolio. An unhedged return represents the return of the bond indices translated to local currency, which is the euro in our case, without hedging. Next the fully hedged returns where 100% of the currency exposure is hedged are shown. Then the return where the constant hedge ratios are found with OLS is presented and lastly the returns with dynamic hedging are shown.

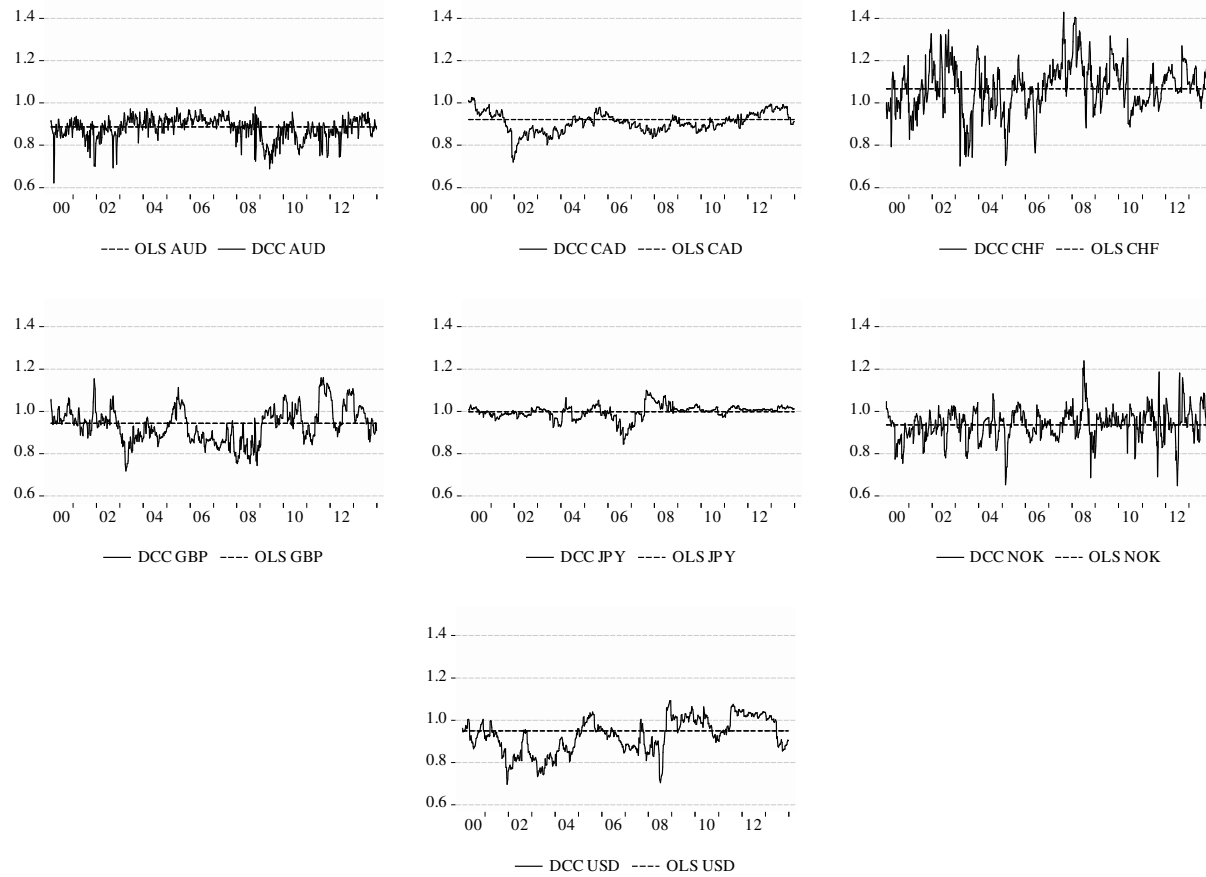


Figure 1: Constant and time-varying hedge ratios for single-instrument portfolios

Note: authors' calculations.

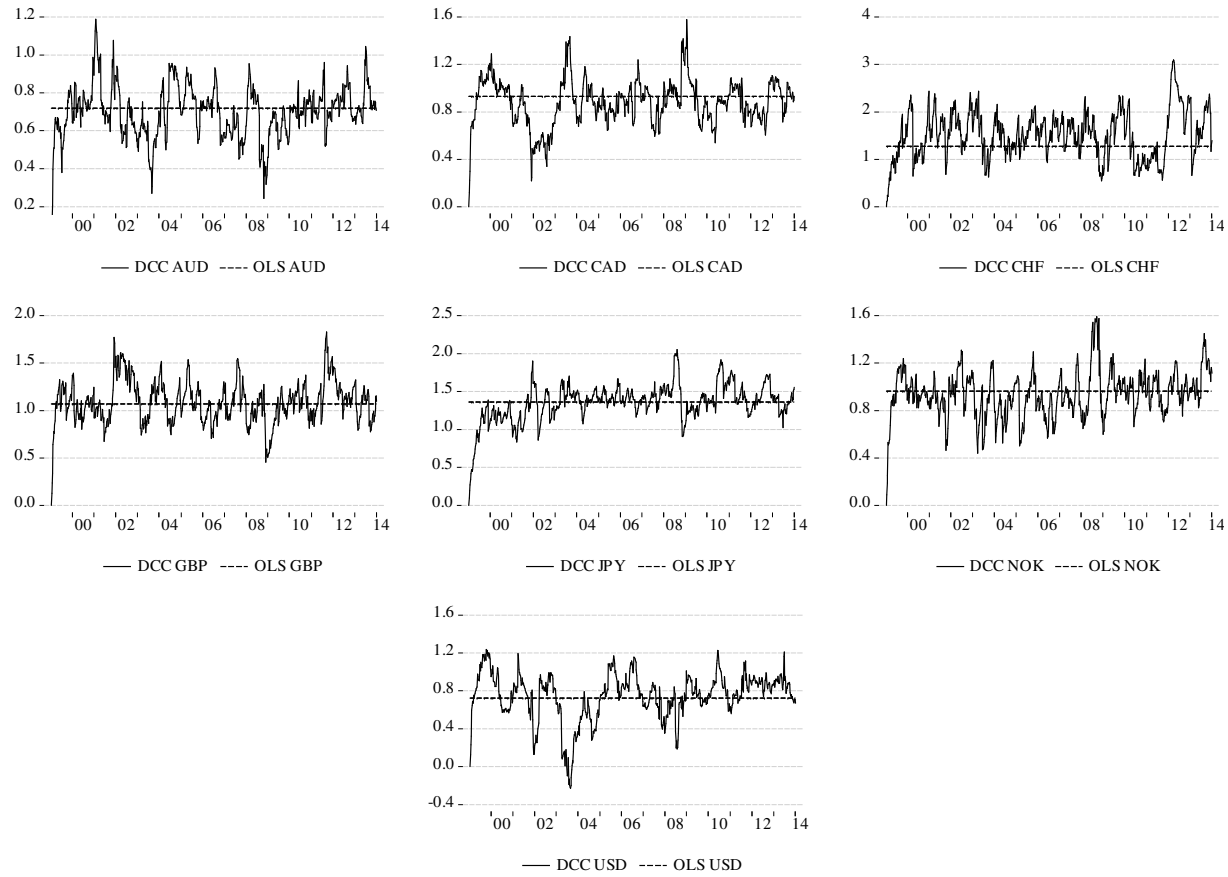


Figure 2: Constant and time-varying hedge ratios for the seven-asset portfolio

Note: authors' calculations.

Table 3 shows the return and risk characteristics of the portfolios. Hedging might reduce the returns but does not necessarily do so. However, the risk-return profiles illustrated by the Sharpe ratios tend to increase with any type of hedging. In addition, we have conducted the Jobson-Korkie (JK) test to compare the Sharpe ratios and the variance ratio test to compare the difference of variance. The test statistics for the Jobson-Korkie test are also shown in Table 3 and Table 4. The Sharpe ratios for the 100% hedged portfolios are significantly higher than those for the unhedged portfolios of single instruments, with the exception of those for the Norwegian krone (for which the difference is not statistically significant, but still positive) and the Australian dollar. The same applies for the equally weighted portfolio.

Table 3: Unhedged and fully hedged portfolio returns

<i>Unhedged return (RU)</i>								
	AUD	CAD	CHF	UK	JPY	NOK	USA	portfolio
Mean	0.140%	0.123%	0.092%	0.076%	0.026%	0.102%	0.079%	0.091%
Std. dev.	1.465%	1.373%	0.914%	1.166%	1.797%	1.002%	1.440%	0.853%
Risk/return	0.096	0.089	0.100	0.065	0.014	0.102	0.055	0.108
<i>100 % hedged return (RH)</i>								
	AUD	CAD	CHF	UK	JPY	NOK	USA	portfolio
Mean	0.044%	0.082%	0.078%	0.069%	0.026%	0.066%	0.086%	0.064%
Std. dev.	0.615%	0.409%	0.285%	0.389%	0.196%	0.434%	0.480%	0.291%
Risk/return	0.071	0.199	0.274	0.178	0.131	0.151	0.180	0.221
Variance ratio test								
RU vs. RH	5.67*	11.24*	10.25*	8.97*	84.43*	5.34*	8.99*	8.59*
Sharpe ratio test								
RU vs. RH	-0.52	2.18**	4.68**	2.42**	2.41**	1.16	2.72**	2.59**

Source: Bloomberg; authors' calculations.

Note: For the variance ratio test, the test statistics are shown. \* indicates the rejection of the null hypothesis of equal variances. For the equality of the Sharpe ratio tests of Jobson and Korkie (1981), the test statistics are shown. \*\* indicates statistical significance at the 5% level.

Regarding the optimal hedging techniques (see Table 4), constant optimal unconstrained hedge ratios produce portfolio Sharpe ratios which significantly outperform those of the unhedged portfolios with the exception of the ratios for the Australian dollar and Norwegian krone. The constant hedge ratio portfolio Sharpe ratio also outperforms the Sharpe ratios of the fully hedged portfolio and the Australian dollar portfolio.

Table 4: Portfolio returns with optimal hedge ratios

<i>Optimal hedge with OLS (OLS)</i>								
	AUD	CAD	CHF	UK	JPY	NOK	USA	portfolio
Mean	0.055%	0.085%	0.077%	0.070%	0.026%	0.068%	0.086%	0.068%
Std. dev.	0.591%	0.394%	0.280%	0.384%	0.196%	0.430%	0.475%	0.267%
Risk/return	0.093	0.215	0.276	0.181	0.131	0.158	0.181	0.255
Variance ratio test								
vs. RU	6.15*	12.14*	10.63*	9.24*	84.44*	5.44*	9.20*	10.24*
vs. RH	1.08	1.08	1.04	1.03	1.00	1.02	1.02	1.19*
vs. DCC	1.02	1.09	1.07	1.04	1.27*	1.06	1.05	1.08
Sharpe ratio test								
vs. RU	-0.08	2.92**	4.10**	2.78**	2.43**	1.46	3.02**	3.44**
vs. RH	2.11**	1.58	0.26	0.59	0.03	1.35	0.24	2.21**
vs. DCC	-1.76	-0.68	0.53	0.46	0.29	-0.20	0.09	-0.15
<i>Optimal hedge with time-varying hedge ratios (DCC)</i>								
	AUD	CAD	CHF	UK	JPY	NOK	USA	portfolio
Mean	0.061%	0.092%	0.079%	0.069%	0.027%	0.070%	0.087%	0.071%
Std. dev.	0.597%	0.412%	0.290%	0.391%	0.220%	0.442%	0.486%	0.276%
Risk/return	0.102	0.223	0.269	0.176	0.123	0.160	0.180	0.257
Variance ratio test								
vs. RU	6.02*	11.10*	9.94*	8.90*	66.44*	5.15*	8.77*	9.52*
vs. RH	1.06	0.99	0.97	0.99	0.79*	0.96	0.97	1.11*
Sharpe ratio test								
vs. RU	0.17	3.19**	3.95**	2.74**	2.32**	1.52	3.09**	3.52**
vs. RH	2.48**	1.44	-0.35	-0.12	-0.29	0.89	0.01	1.60
<i>Optimal hedge with OLS constrained</i>								
	AUD	CAD	CHF	UK	JPY	NOK	USA	portfolio
Mean								0.069%
Std. dev.		OLS		RH			OLS	0.281%
Risk/return								0.247
Variance ratio test								
vs. RU								9.24*
vs. RH		OLS		RH			OLS	1.08
Sharpe ratio test								
vs. RU								3.63**
vs. RH		OLS		RH			OLS	2.59**
<i>Optimal hedge with time-varying constrained hedge ratios</i>								
	AUD	CAD	CHF	UK	JPY	NOK	USA	portfolio
Mean	0.061%	0.092%	0.080%	0.070%	0.028%	0.070%	0.088%	0.070%
Std. dev.	0.597%	0.412%	0.293%	0.390%	0.220%	0.440%	0.485%	0.296%
Risk/return	0.102	0.223	0.272	0.176	0.122	0.159	0.180	0.238
Variance ratio test								
vs. RU	6.02*	11.10*	9.70*	8.95*	66.96*	5.19*	8.80*	8.32*
vs. RH	1.06	0.99	0.95	0.997	0.79*	0.97	0.98	0.97
Sharpe ratio test								
vs. RU	0.17	3.19**	4.76**	2.80**	2.42**	1.52	3.15**	4.14**
vs. RH	2.48**	1.43	-0.22	-0.16	-0.32	0.84	-0.01	0.84

Source: Bloomberg; authors' calculations. Note: For the variance ratio test, the test statistics are shown. \* indicates the rejection of the null hypothesis of equal variances. For the equality of the Sharpe ratio tests of Jobson and Korkie (1981), the test statistics are shown. \*\* indicates statistical significance at the 5% level. Regarding the results with constrained optimization by single instrument in the portfolio it must be noted that the short-selling restriction was applied only on the Swiss franc and, as noted, with the short-selling restriction the optimal hedge ratio was then 100% which corresponds to the full hedge. For the other currencies, the estimated hedge ratios were below 100% and thus the results would remain the same as the OLS results. The results are different for the portfolio of multiple assets as reported above.

In the same vein, the time-varying hedge ratio produces a portfolio Sharpe ratio which is significantly higher than that for the unhedged one. The Australian dollar and Norwegian krone remain the exceptions even though the numerical values of the Sharpe ratios are higher. However, we cannot conclude that the dynamic hedge ratios are superior to the fully hedged results except for those with the Australian dollar.

If we compare the performance of the constant and time-varying hedge ratios in terms of Sharpe ratios, we can see that there is no significant difference. Only for the Japanese yen do we find significant difference in the Sharpe ratios.

We also see a numerical reduction in volatility in all the hedged portfolios compared to the unhedged portfolio returns. We see that the volatility of the portfolio of multiple assets is lower than the volatilities of the single bond portfolio, which once again confirms the benefits of diversification. The equally weighted unhedged portfolio returns are less volatile than the single instrument investments, but are still around three times more volatile than the hedged portfolios. However, we also see from our results that any kind of hedging reduces the risk of the portfolios even more, at least numerically. Thus it can be concluded that diversification works but it does not fully deal with the currency risk associated with the foreign bond portfolios. Hedging also improves the risk-return profile significantly.

The variances of the unhedged portfolios for both single and multiple assets are indeed significantly higher than those for the fully hedged portfolios. The unhedged multiple asset portfolio risk is also significantly higher than the risks of the other four hedged portfolios. Comparing the variances of the optimal hedge ratio portfolios, we cannot find a conclusive result as to which ratio is significantly better than the others. We can only conclude that for the multi-asset portfolio, the constant optimal hedge ratio produces a lower portfolio variance than the fully hedged portfolio.

We can conclude that for the Sharpe ratios, neither full hedging nor zero hedging is optimal. The risk-return trade-off can significantly improve with optimal hedging but the results for volatility are not so clear-cut. It is clear that risks can be reduced by hedging even though the choice between the 100% and optimal hedge ratios is not straightforward. It is clear that in a real situation the choice will be governed by the restrictions which apply for the investor, such as short-selling restrictions.

## 6. Discussion

The current paper analyses whether and to what extent foreign bond investments should be hedged in order to minimize the variance of the overall portfolio. We have taken the perspective of the euro area investor and have used a weekly frequency. Starting with a comparison of unhedged and fully hedged portfolio risk-return profiles, we then proceed to find constant optimal hedge ratios using the conventional minimum variance framework, under which the investor is risk adverse and wishes to minimize the volatility of the portfolio. Due to the time varying nature of the asset returns, the analysis is expanded with the use of the multivariate time series DCC-GARCH approach. We have also taken account of the possible short selling restrictions which could be applied and have analyzed the questions under observation for both single bond and multiple bond portfolios.

We show that portfolios of foreign bond investments are sensitive to foreign currency risk and these risks can only partially be mitigated in multiple asset portfolios. In line with the previous literature on international fixed income portfolios, we found that any kind of hedging improves the risk-return profiles in our sample significantly and the constant hedge ratio tends to dominate slightly. Unhedged positions perform the worst in terms of the volatility of the domestic-currency value of the portfolios.

A deeper look at the resulting optimal hedge ratios shows that on average, the ratios differ depending on whether they are in the portfolio or found for each of the single instruments. The results from the empirical studies which conduct the analysis for portfolios with only one or two assets cannot be applied to the portfolios, as all of the variance-covariance relationships must be accounted for. The comparison between the fully hedged and the optimally hedged portfolios shows that, at least in the sample analysed here, full hedging gives a worse result than optimal hedging, in terms of both the volatility of the return and the Sharpe ratio. It is not possible to reach a conclusive result as to whether the constant or time-varying hedge ratios should be used. The minimum-variance framework seems to work better for the portfolio even if a short-selling restriction is applied. The numerical Sharpe ratios are better with the dynamic approach but the constant hedge ratios produce less variance.

There are some caveats that should be stressed here. First of all, the horizon at which hedging is performed may change the conclusions of our analysis. Some of the articles reviewed in Section 2 assume that hedging is performed daily, which is probably not what generally happens in practice. We assume a weekly horizon, but it would be useful to perform the same analysis with monthly or quarterly hedging to assess how the results can change.

Second, a different portfolio structure could also change the results. In particular, there are two changes that can have an impact. First, investors often have a home bias in their portfolio, tending to have a higher exposure to local securities. Adding euro area government bonds to the portfolio could change the correlation structure of the portfolio, thus changing the conclusion on hedging. Second, adding to the portfolio a riskier asset such as equities, which tend to have higher volatility but lower correlation with the asset classes analysed here, could also have an impact on the results.

Some care needs to be taken when the results are applied to real portfolios. We have not studied the validity of our results with reference to the investment horizon. Whether these results would remain relevant with an investment horizon of, for example, five to ten years remains a topic for further research. Furthermore, the analysis so far has been backward looking; the performance of the hedge ratios out of the sample are at least as important if not more important. We have used a somewhat arbitrary equally weighted portfolio for our analyses instead of an optimized structure, while in reality the bond portfolio construction can be subject to several policy considerations. A comparison of several portfolios with different weights should give a clearer picture about the validity of these results. Nevertheless, we remain convinced about the main conclusions, which advocate the use of hedging in the short term, in particular optimal hedging to gain the best risk-return trade-off.



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