Eesti Pank
Bank of Estonia

An Estimated Dynamic Stochastic General Equilibrium Model for Estonia

Paolo Gelain and Dmitry Kulikov

Working Paper Series
5/2009

The Working Paper is available on the Eesti Pank web site at:
www.bankofestonia.info/pub/en/dokumendid/publikatsioonid/seeriad/uuringud/

For information about subscription call: +372 668 0998; Fax: +372 668 0954
e-mail: publications@epbe.ee

ISBN 978-9949-404-80-3
ISSN 1406-7161

An Estimated Dynamic Stochastic General Equilibrium Model for Estonia

Paolo Gelain and Dmitry Kulikov*

Abstract

This paper presents an estimated open economy dynamic stochastic general equilibrium model for Estonia. The model is designed to highlight the main driving forces behind the Estonian business cycle and to understand how euro area economic shocks and its monetary policy affect the small open economy of Estonia. The model described in this paper is a two-area DSGE model incorporating New Keynesian features such as nominal price and wage rigidity, variable capital utilization, investment adjustment costs, as well as other typical features — both for the domestic and euro area part of the model. It is rich in structural shocks such as technology, consumption preference, mark-up, etc. The model is estimated by Bayesian techniques using a quarterly data sample that covers main macroeconomic aggregates of Estonia and the euro area. The ultimate goal of the new model is for it to be used in simulation exercises, policy advice and forecasting at the Bank of Estonia.

JEL Code: E4, E5

Keywords: monatary policy, New Keynesian models, small open economy, Bayesian statistical inference

Author's e-mail address: dmitry.kulikov@epbe.ee

*The principal work on the model was completed during Paolo Gelain's stay as a visiting researcher at Eesti Pank from June to September 2008. We also wish to thank seminar participants at Eesti Pank and Stockholm School of Economics in Riga, as well as participants of the Annual Estonian Economic Association Meeting in Toila, SMYE Conference in Istanbul, and ECEE Conference in Tallinn for many helpful suggestions and comments about the paper. All remaining mistakes and omissions are our own.

Non-technical summary

This paper outlines the theoretical foundations and presents main empirical results of the first version of an open economy dynamic stochastic general equilibrium (DSGE) model for Estonia developed by the Bank of Estonia. One of the main goals of building a DSGE model for Estonia is to use it to understand the monetary policy and export-import linkages between a small open economy of Estonia and the much bigger euro area economy. The list of other potential uses of the new model includes simulation exercises, policy advice and forecasting of the main macroeconomic aggregates.

The Bank of Estonia's DSGE model described in this paper is a New Keynesian DSGE model which incorporates many important features that are found to be essential for reproducing the complex dynamics and persistence of the real-world macroeconomic time series. It incorporates the key ingredients that are needed to effectively describe the functioning of the Estonian economy:

- The currency board regime, free capital mobility and resulting lack of an independent monetary policy conducted by the national central bank. The monetary policy of Estonia is effectively imported from the European Central Bank and therefore depends on the euro area business cycle. The spread between domestic and euro area interest rates is the key to understand the macroeconomic developments in Estonia over the last decade;

- The Estonian economy is a textbook example of a small open economy in terms of its openness to foreign trade as well. The impact of the euro area business cycle on the domestic economy of Estonia via mutual trade linkages is very pronounced;

- Real and nominal convergence still features prominently in the main macroeconomic aggregates of Estonia. However, the first version of the Bank of Estonia's DSGE model presented in this paper is specified for the business cycle frequency only, where the long-run dynamics of the main observables is filtered out. Future revisions of the model will address this issue with due care.

A unique feature of the Bank of Estonia's DSGE model is the inclusion of a fully specified, partly estimated DSGE model for the euro area. The economy of Estonia is considered to be a small open economy on the fringes of the euro area — its main trading partner and *de facto* implement of Estonia's monetary policy due to the currency board arrangement and free capital mobility

between the two economies. The euro area part of the model is a fully articulated New Keynesian DSGE model of Smets and Wouters (2003), subject to its own set of seven structural shocks, that is designed to reproduce the monetary policy conducted by the European Central Bank, and to act as a foreign market for Estonian exports and imports. The two-area setup of the Bank of Estonia's DSGE model allows for meaningful simulations of the impact that the euro area monetary policy has on the small open economy of Estonia. The anticipated integration of Estonia into the common currency area makes a thorough understanding of these effects particularly important.

The empirical part of this paper deals with Bayesian estimation of the new model. Out of 59 structural parameters in the Bank of Estonia's DSGE model, 52 are estimated using a quarterly data sample consisting of 16 macroeconomic series for Estonia and the euro area. The statistical estimates of the main parameters are mostly in line with previous studies for Estonia, whenever a direct comparison can be made. It is also worth mentioning that the net foreign asset position of Estonia is found to be an economically significant determinant of the interest rate spread, but the empirical results suggest that other explanatory factors may also be warranted.

The empirical relevance of structural shocks is assessed using variance decomposition of the main endogenous variables in the model. A consumption preference shock and two technology shocks are found to be the most important contributors to the variability of the main domestic macroeconomic aggregates. Euro area shocks also play a very prominent role in driving the dynamics of Estonian macroeconomic series. Among the most significant shocks impacting on the Estonian economy are the euro area price and wage mark-up shocks.

As mentioned previously, the first version of the Bank of Estonia's DSGE model in this paper is focused on the business cycle frequency of the main Estonian macroeconomic aggregates, leaving their long-run trends aside. Future developments of the model are likely to incorporate the long-run dynamics as well, considering that Estonia is still experiencing the effects of real and nominal convergence as it catches up with the developed euro area economies. Other potential future extensions of the model include incorporation of the financial sector with the associated frictions, integration of the housing sector together with collateral-constrained households, and expansion of the government sector.

Contents

1. Introduction

This paper works out the theoretical foundations and reports main empirical findings for the first version of an open economy dynamic stochastic general equilibrium model for Estonia developed by the Bank of Estonia. One of the main goals of building a DSGE model of the Estonian economy is to use it to understand the monetary policy and export-import linkages between a small open economy of Estonia and the much larger euro area economy. The list of other potential tasks for the Bank of Estonia's dynamic stochastic general equilibrium model, henceforth abbreviated as EP DSGE, includes simulation exercises, policy advice and forecasting of the main macroeconomic aggregates.

For now, all these tasks are carried out by EMMA model (see Kattai, 2005). The EMMA model is a traditional medium-scale backward-looking macroeconomic model estimated on an equation-by-equation basis. It incorporates a number of theory-based restrictions, but unlike a typical DSGE model it is not derived from the ground up using the utility and profit maximization framework of modern macroeconomics.

Recently, a new breed of micro-founded DSGE models that incorporate a large number of structural shocks, nominal and real rigidities, and other features necessary to describe the persistence of real-world macroeconomic time series has received a lot of attention by the leading monetary policy institutions around the world; refer to Tovar (2008) for a recent survey of DSGE modeling at central banks. These models became possible thanks to advances in macroeconomic theory, offering an advantage over the traditional backward-looking models in terms of clear interpretations of the main relationships among the forward-looking economic agents that are subject to the uncertainty stemming from a large number of well-motivated structural disturbances. In addition, the newly found popularity of DSGE models in many central banks comes from recent developments in powerful computational methods that permit Bayesian statistical inference for a large number of structural parameters from real-world macroeconomic data.

Likewise, the first version of the EP DSGE model presented in this paper is a step toward eventually phasing out the EMMA model at the Bank of Estonia as the main tool for simulation of different macroeconomic scenarios and policy advice. However, a substantial amount of work remains to be done before the new model is sufficiently refined and ready to be used by policy makers.

A DSGE approach to modeling the Estonian economy has been previously attempted in Colantoni (2007) and Lendvai and Roeger (2008). Colantoni (2007) estimates a two-area DSGE model using Estonian macroeconomic data with the goal of studying the interest rate channel of monetary policy transmission between Estonia and the euro area. While the EP DSGE model has similar objectives, its structure has been refined to better reflect the existing monetary policy regime and to add the foreign trade channel to the interaction between Estonia and euro area. Another difference between Colantoni (2007) and this paper is a more careful empirical implementation of the model. The second paper by Lendvai and Roeger (2008) calibrates an open economy DSGE model with several types of households, the housing sector and separate tradable and non-tradable production sectors in order to assess the relative importance of productivity growth and credit expansion in driving the long-run trends of the main Estonian macroeconomic aggregates over the last decade. In contrast to Lendvai and Roeger (2008), where a specific simulation exercise is carried out to understand the long-run trends, the EP DSGE model is focused on the effects of the euro area's monetary policy and export-import linkages on the economy of Estonia at the business cycle frequency.

The EP DSGE model described in this paper is a New Keynesian DSGE model which incorporates many important features that are found to be essential for describing the complex dynamics and persistence of real-world macroeconomic time series. The key references for the model are papers by Smets and Wouters (2003), Christiano, Eichenbaum and Evans (2005) and Adolfson et al. (2007a). Specifically, the EP DSGE model incorporates external consumption habits, investment adjustment costs, price and wage rigidities and indexation to past inflation, and variable capital utilization.

In addition to these frictions, the model contains nine structural shocks that determine dynamics of the Estonian economy. Among the fundamental shocks are the production and investment-specific technology innovations, a consumption preference shock, and a fiscal policy shock. The domestic "cost-push" shocks include a stochastic price mark-up in the production sector and a wage mark-up in the labour demand function. Interactions between the economies of Estonia and euro area in the model are driven by stochastic mark-up shocks in the export and import sectors, as well as an idiosyncratic risk premium shock in the equation linking domestic and euro area interest rates.

The open economy aspect of the EP DSGE model is based on the paper by Adolfson et al. (2007a). In particular, exporting and importing firms in the model operate by selling differentiated consumption good to foreign and domestic markets subject to the local currency price stickiness and indexation to past inflation. In contrast to Adolfson et al. (2007a), where both consumption and investment goods are traded, the economies of Estonia and euro area in the EP DSGE model trade in the final consumption good only. This simplification is introduced due to the unavailability of suitably disaggregated import and export price indices in the Estonian foreign trade statistics. Other differences from Adolfson et al. (2007a) include the omission of the unit root technology shock in favor of a stationary one, missing working capital channel of the monetary policy, a much less articulated modeling of the government sector, as well as the inclusion of a fully specified, partly estimated DSGE model for the euro area.

The latter feature of the EP DSGE model is particularly important considering the design goals and prospective use of the model at the Bank of Estonia. The economy of Estonia is considered to be a small open economy on the fringes of the euro area — its main trading partner and *de facto* implement of Estonia's monetary policy due to the currency board arrangement and free capital mobility between the two economies. The euro area part of the EP DSGE is a fully articulated New Keynesian DSGE model of Smets and Wouters (2003), subject to its own set of seven structural shocks, which is designed to reproduce monetary policy conducted by the European Central Bank, and to act as a foreign market for Estonian exports and imports. The two-area setup of the EP DSGE model allows for meaningful simulations of the euro area's monetary policy effects on the domestic economy of Estonia. The anticipated integration of the Estonian economy into the common currency area makes a thorough understanding of these effects particularly important.

The empirical results obtained and reported in this paper can be considered satisfactory for the first version of the model. The statistical estimates of the main structural parameters are largely in line with previous studies for Estonia when a direct comparison can be made. However, few areas still await an improvement in the future versions of the model. The external sector is of particular concern, where both the dynamics of the trade linkages with the euro area as well as the role of net foreign assets in picking up the spread between domestic and euro area interest rates need further examination.

The paper is structured as follows: Section 2 provides a short summary of the main building blocks of the EP DSGE model, at the same time avoiding excessive technical details. Section 3 and 4 describe the key equations of the model pertaining to the economies of Estonia and euro area. The log-linearized versions of these equations are reported in Appendices 8.2 and 8.3. An overview of the statistical methodology, data series, prior distributions and calibrated parameters is given in Section 5. The main empirical results are discussed in detail in Section 6. Conclusion summarizes the main findings of the paper.

2. An Overview of the EP DSGE Model

The EP DSGE model presented in this paper takes into account the following key features of the Estonian economy:

- The currency board regime, free capital mobility and resulting lack of an independent monetary policy conducted by the national central bank. The monetary policy of Estonia is effectively imported from the ECB and therefore depends on the euro area's business cycle.[1] The spread between domestic and euro area interest rates is the key to understanding the macroeconomic developments in Estonia over the last decade;
- The Estonian economy is a textbook example of a small open economy in terms of its openness to foreign trade as well. The impact of the euro area's business cycle on the domestic economy of Estonia via mutual trade links is very important;
- Real and nominal convergence still features prominently in the main macroeconomic aggregates of Estonia. However, the first version of the EP DSGE model reported in this paper is specified for the business cycle frequency only, and filters out the long-run dynamics contained in the empirical data.[2]

Figure 1 previews the main building blocks and resource flows inside the EP DSGE model. It is a two-area DSGE model, consisting of a small open economy DSGE model for Estonia and a large closed economy DSGE model for the euro area. The two parts are linked through the monetary policy channel — one way from the euro area to Estonia — and by the export-import flows, where the euro area economy serves as a source of imports to the home economy of Estonia and generates demand for Estonian exports.[3] Foreign

[1] Prior to re-pegging of the Estonian Kroon to the euro in 1999, it was fixed to the Deutsche Mark at the rate of 1 DM = 8 EEK. During the second half of the 1990s, the Estonian banking system was still not completely integrated with the European and Scandinavian ones. The Asian financial crisis of 1997 and the subsequent Russian financial crisis of 1998 have changed the landscape of the Estonian banking sector, effectively putting all major Estonian banks into the hands of Scandinavian owners. Since then, the spreads between domestic and euro area interest rates have narrowed dramatically.

[2] The future versions of the model are likely to address this issue by incorporating unit root technology and suitable steady state inflation dynamics.

[3] The breakdown of Estonian trade statistics in 2008 reveals that 70% of foreign trade takes place with EU countries. However, the share of euro area countries in foreign trade is around 25% because many of Estonia's major trading partners in the Baltic Sea region, such as Latvia, Lithuania, Sweden, Denmark and Poland, are not euro area members. Since these countries are themselves highly open to euro area trade, the assumption of the EP DSGE model about import-export trading links with the euro area is a reasonable approximation.

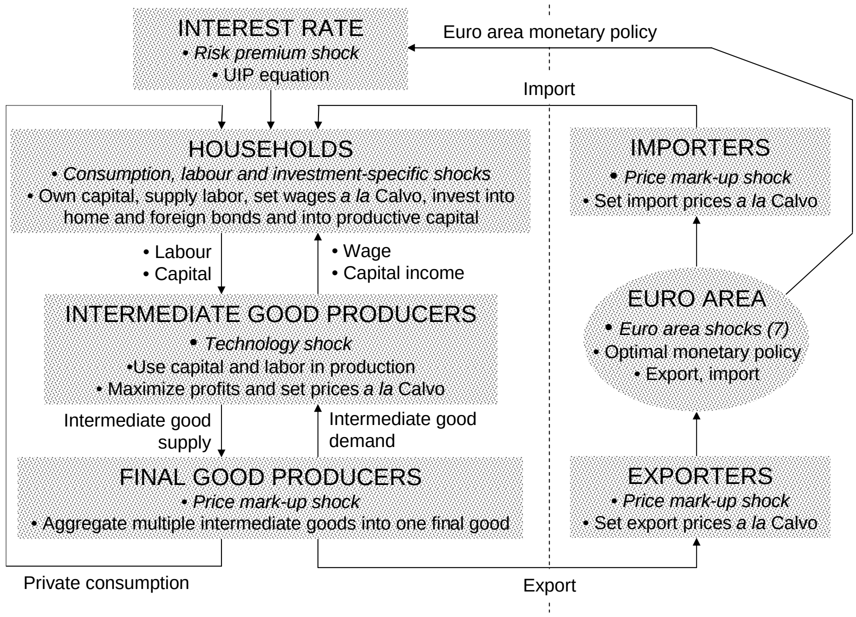

Figure 1: A diagram of the EP DSGE model

trade with the euro area is assumed to be in terms of the composite final consumption good only.

The Estonian part of the EP DSGE is a fairy typical small open economy DSGE model that is similar to Adolfson et al. (2007a). There are 25 state variables and 9 structural shocks.[4] The main building blocks of the Estonian part of the model can be shortly summarized as follows:

- Households own labour and capital, optimize their consumption and supply of working hours across time, set wages in the Calvo (1983) manner subject to labour demand from the labour aggregator, and invest in domestic and foreign bonds as well as productive capital;
- Firms are of four types: final good producers operating in a perfectly competitive market, monopolistically competitive domestic intermediate good producers that set prices in the Calvo (1983) manner, and import and export firms that set prices of differentiated consumption goods in the Calvo (1983) manner;
- The government sector is assumed to follow a balanced budged fiscal policy driven by an exogenous government spending shock;
- Domestic nominal interest rate is linked to the euro area interest rate via the modified uncovered interest rate parity (UIP) condition $R_t^n =$

[4]Appendix 8.2 reports all final model equations in log-linear form.

$\Omega(FA_t, \epsilon_t^{\text{risk}})\, R_t^{n,*}$. The currency board regime is manifested in the absence of an exchange rate risk in this equation.[5] Instead, an idiosyncratic part of the interest rate spread is picked up by ϵ_t^{risk}.

The euro area part of the EP DSGE is a partly estimated — partly calibrated version of the Smets and Wouters (2003) closed economy DSGE model with 13 state variables and 7 structural shocks.[6] So called "deep parameters" of the model are calibrated according to the results in Smets and Wouters (2003), whereas parameters related to the structural shocks are estimated jointly with the Estonian economy part of the model.

3. Key Equations: The Estonian Economy

3.1. Households

Household $i \in [0,1]$ maximizes its inter-temporal utility function by choosing how much to consume, $\{C_t^i : t \geq 0\}$; how much to invest today in order to build the capital that will be used in production tomorrow, $\{I_t^i : t \geq 0\}$; the hours it wants to work, $\{L_t^i : t \geq 0\}$; the utilization rate of capital, $\{z_t^i : t \geq 0\}$; how much capital to lend to the firms, $\{K_t^i : t \geq 0\}$; and how many domestic, $\{B_t^i : t \geq 0\}$, and euro area, $\{B_t^{i,*} : t \geq 0\}$, bonds to hold:[7]

$$\max_{\{C_t^i, I_t^i, L_t^i, z_t^i, K_t^i, B_t^i, B_t^{i,*} : t \geq 0\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \varepsilon_t^{\beta} \Big[\frac{1}{1-\sigma_c} (C_t^i - h\, C_{t-1})^{1-\sigma_c} - \frac{1}{1+\sigma_l} (L_t^i)^{1+\sigma_l} \Big],$$

where $\log \varepsilon_t^{\beta} = \rho_\beta \log \varepsilon_{t-1}^{\beta} + u_t^{\beta}$, $u_t^{\beta} \sim \text{WN}(0, \sigma_\beta^2)$ is the preference shock. Households consumption behavior is characterized by the external habit: each household in the economy derives a positive utility from consumption in pe-

[5] In fact, the institutional arrangement of the 17-year-old currency board system in Estonia rules out the possibility of a unilateral Euro peg rate change by the central bank of Estonia. All such changes must be enacted by the national parliament and therefore are likely to take time before coming into effect. The institutional structure of the currency board in Estonia therefore prevents unexpected and unannounced changes of the nominal exchange rate.

[6] Appendix 8.3 reports all final model equations in log-linear form.

[7] Households' domestic bond holdings B_t^i in the Estonian economy part of the EP DSGE model can be thought of as a proxy for per capita net short-term saving/borrowing by residents in Estonian banks; ditto the euro area bonds $B_t^{i,*}$ in foreign banks. There is no market for short-term government obligations in Estonia, and almost all the financing needs of Estonian households and firms are met by the banking sector. The first version of the EP DSGE presented in this paper does not explicitly model the banking sector, an omission that is likely to be addressed in the future versions of the model.

riod $t \geq 0$ only if it is able to consume more than a fraction h of the economy-wide per capita consumption at $t-1$. The inverse of the inter-temporal elasticity of substitution in consumption (or equivalently the coefficient of relative risk aversion) and the inverse of the elasticity of work effort with respect to the real wage are denoted by σ_c and σ_l, respectively.

Maximization of the inter-temporal utility function is constrained. Firstly, in each time period $t \geq 0$ every household faces the following budget constraint expressed in real terms:[8]

$$\begin{aligned} &C_t^i + I_t^i + B_t^i + \overline{e}\, B_t^{i,*} = \\ &R_{t-1}^n \frac{B_{t-1}^i}{\pi_t^c} + \Omega(FA_{t-1}, \epsilon_{t-1}^{\text{risk}})\, \overline{e}\, R_{t-1}^{n,*} \frac{B_{t-1}^{i,*}}{\pi_t^c} + \frac{W_t^i}{P_t^c} L_t^i + R_t^k z_t^i K_{t-1}^i - \Psi(z_t^i) K_{t-1}^i \\ &\quad + T_t^i + D_t^i \,, \end{aligned}$$

where R_t^n denotes the gross nominal domestic interest rate, T_t^i are the net transfers, D_t^i are dividends from the final good producers which are assumed to be owned by the households, π_t^c is the gross rate of consumer inflation defined as $\pi_t^c := \frac{P_t^c}{P_{t-1}^c}$, where P_t^c denotes the consumer price index, $\overline{e}$ is the fixed nominal exchange rate, W_t^i is the nominal wage earned by the household, R_t^k is the return on capital, $\Psi(z_t^i)$ captures the cost of capital utilization,[9] and $\Omega(FA_t, \epsilon_t^{\text{risk}})$ is the country specific risk premium function:[10]

$$\log \Omega(FA_t, \epsilon_t^{\text{risk}}) = -\phi_{\text{fa}} FA_t + \log \epsilon_t^{\text{risk}} \,, \tag{1}$$

where $FA_t := \frac{\overline{e} B_t^{*,n}}{P_t^d}$ is the net foreign asset position of the Estonian economy, P_t^d denotes the domestic price index, and $\log \epsilon_t^{\text{risk}} = \rho_{\text{risk}} \log \epsilon_{t-1}^{\text{risk}} + u_t^{\text{risk}}$, $u_t^{\text{risk}} \sim \text{WN}(0, \sigma_{\text{risk}}^2)$ is an idiosyncratic component of the country specific risk.[11] Equation (1) captures imperfect integration of the Estonian economy

---

[8]In nominal terms, the budget constraint is given by:

$$\begin{aligned} &P_t^c C_t^i + I_t^{i,n} + B_t^{i,n} + B_t^{i,*,n} = \\ &R_{t-1}^n B_{t-1}^{i,n} + \Omega(FA_{t-1}, \epsilon_{t-1}^{\text{risk}})\, \overline{e}\, R_{t-1}^{n,*} B_{t-1}^{i,*,n} + W_t L_t^i + R_t^k K_{t-1}^{i,n} + T_t^{i,n} + D_t^{i,n} \,. \end{aligned}$$

After dividing by P_t^c it becomes:

$$C_t^i + I_t^i + B_t^i + B_t^{i,*} = R_{t-1}^n \frac{B_{t-1}^{i,n}}{P_t^c} + \Omega(FA_{t-1}, \epsilon_{t-1}^{\text{risk}})\, \overline{e}\, R_{t-1}^{n,*} \frac{B_{t-1}^{i,*,n}}{P_t^c} + \frac{W_t}{P_t^c} L_t^i + R_t^k K_{t-1}^i + T_t^i + D_t^i \,.$$

Then, multiplying $\frac{B_{t-1}^{i,n}}{P_t^c}$ and $\frac{B_{t-1}^{i,*,n}}{P_t^c}$ terms by $\frac{P_{t-1}^c}{P_{t-1}^c}$, they become $\frac{B_{t-1}^i}{\pi_t^c}$ and $\frac{B_{t-1}^{i,*}}{\pi_t^c}$ respectively, and the expression in the text is obtained.

[9]Function Ψ satisfies $\Psi(1) = 0$, that is needed for mathematical convenience in steady-state computations and log-linearization.

[10]Assumptions about the steady state behavior of the risk premium function are given in Appendix 8.4.

[11]See Lundvik (1992) and Benigno (2001).

into the euro area financial markets. The higher the indebtedness *vis-à-vis* the rest of the world, the higher the risk of a default and consequently the higher the risk premium the country has to pay over the euro area interest rate. In addition, the risk premium is needed to ensure a well-defined steady state in the model (see Schmitt-Grohè and Uribe, 2003).

Secondly, the capital stock in the economy is owned by the households, and every household faces the following capital accumulation equation in each time period $t \geq 0$:

$$K_t^i = (1-\delta)K_{t-1}^i + \left[1 - S\left(\frac{I_t^i}{I_{t-1}^i}\right)\right] I_t^i\, \varepsilon_t^x\,, \tag{2}$$

where δ is the depreciation rate of capital, $\log \varepsilon_t^x = \rho_x \log \varepsilon_{t-1}^x + u_t^x$, $u_t^x \sim \mathrm{WN}(0, \sigma_x^2)$ is a stationary investment-specific technology shock common across all households in the economy, and $S(\frac{I_t^i}{I_{t-1}^i})$ is the investment adjustment cost function.[12]

The Lagrangian equation is as follows:

$$\max_{\{C_t^i, I_t^i, L_t^i, z_t^i, K_t^i, B_t^i, B_t^{i,*}\,:\, t\geq 0\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ell(C_t^i, I_t^i, L_t^i, z_t^i, K_t^i, B_t^i, B_t^{i,*})\,, \tag{3}$$

where:

$$\begin{aligned}
\ell(C_t^i, I_t^i, L_t^i, z_t^i, K_t^i, B_t^i, B_t^{i,*}) &:= \frac{\varepsilon_t^\beta}{1-\sigma_c}(C_t^i - h\, C_{t-1})^{1-\sigma_c} - \frac{\varepsilon_t^\beta}{1+\sigma_l}(L_t^i)^{1+\sigma_l} \\
&+ \lambda_t \left[\begin{array}{c} R_{t-1}^n \frac{B_{t-1}^i}{\pi_t^c} + \Omega(FA_{t-1}, \epsilon_{t-1}^{\text{risk}})\, \overline{e}\, R_{t-1}^{n,*} \frac{B_{t-1}^{i,*}}{\pi_t^c} + \frac{W_t^i}{P_t^c} L_t^i + R_t^k z_t^i K_{t-1}^i \\ -\Psi(z_t^i) K_{t-1}^i + T_t^i + D_t^i - C_t^i - I_t^i - B_t^i - \overline{e}\, B_t^{i,*} \end{array} \right] \\
&+ Q_t \left[(1-\delta) K_{t-1}^i + \left[1 - S\left(\frac{I_t^i}{I_{t-1}^i}\right)\right] I_t^i \varepsilon_t^x - K_t^i \right].
\end{aligned}$$

[12] Function S satisfies the following properties (see Christiano et al., 2005): $S(1) = S'(1) = 0$ and $S''(1) > 0$.

The sequence of first-order conditions, one for each $t \geq 0$, is given by:[13]

$$
\begin{aligned}
(\partial C_t) \quad & \varepsilon_t^\beta (C_t - h\, C_{t-1})^{-\sigma_c} = \lambda_t \,, & (4)\\
(\partial I_t) \quad & Q_t\, \varepsilon_t^x \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right) \cdot \frac{I_t}{I_{t-1}}\right] & \\
& \qquad + \beta\, \mathbb{E}_t\, Q_{t+1}\, \varepsilon_{t+1}^x\, S'\left(\frac{I_{t+1}}{I_t}\right) \cdot \left(\frac{I_{t+1}}{I_t}\right)^2 = \lambda_t \,, & (5)\\
(\partial z_t) \quad & R_t^k = \Psi'(z_t) \,, & (6)\\
(\partial K_t) \quad & Q_t = \beta\, \mathbb{E}_t\, \lambda_{t+1} [z_{t+1} R_{t+1}^k - \Psi(z_{t+1})] + \beta(1-\delta)\, \mathbb{E}_t\, Q_{t+1} \,, & (7)\\
(\partial B_t) \quad & \beta\, \mathbb{E}_t\, \lambda_{t+1}\, R_t^n \frac{1}{\pi_{t+1}^c} = \lambda_t \,, & (8)\\
(\partial B_t^*) \quad & \beta\, \mathbb{E}_t\, \lambda_{t+1}\, \Omega(FA_t, \epsilon_t^{\text{risk}})\, R_t^{n,*} \frac{1}{\pi_{t+1}^c} = \lambda_t \,. & (9)
\end{aligned}
$$

The first-order condition with respect to L_t is derived in the next section because households are assumed to supply labour monopolistically. In the special case of a competitive labour market, the sequence of first order conditions that determine an optimal labour effort is given by:

$$
(\partial L_t) \quad - \varepsilon_t^\beta L_t^{\sigma_l} + \lambda_t \frac{W_t}{P_t^c} = 0 \quad \forall t \geq 0 \,. \tag{10}
$$

Equation (4) is the usual consumption Euler equation. The ratio of two Euler equations in time periods t and $t+1$ must satisfy:

$$
\mathbb{E}_t \frac{\lambda_t}{\lambda_{t+1}} = \mathbb{E}_t \frac{\varepsilon_t^\beta (C_t - h\, C_{t-1})^{-\sigma_c}}{\varepsilon_{t+1}^\beta (C_{t+1} - h\, C_t)^{-\sigma_c}} \,. \tag{11}
$$

From Equation (8) follows that:

$$
\mathbb{E}_t \frac{\lambda_t}{\lambda_{t+1}} = \beta\, \mathbb{E}_t\, R_t^n \frac{1}{\pi_{t+1}^c} \,.
$$

Combining the previous equation with (11) leads to the optimal consumption dynamics, given in the log-linear form by:

$$
\widehat{c}_t = \frac{h}{1+h} \widehat{c}_{t-1} + \frac{1}{1+h}\, \mathbb{E}_t\, \widehat{c}_{t+1} - \frac{1-h}{\sigma_c(1+h)} [\widehat{r}_t^n - \mathbb{E}_t\, \widehat{\pi}_{t+1}^c] + \widehat{\varepsilon}_t^\beta \,.
$$

Equations (5) and (7) may be re-written to define the marginal Tobin's Q as the ratio of the two Lagrangian multipliers $q_t = \frac{Q_t}{\lambda_t}$, or more loosely the

[13]The index i is skipped because the decentralized solution is the same as the centralized one; hence, all the first-order conditions are the same across the households in the economy.

value of installed capital in terms of its replacement cost. They become respectively:[14]

$$
\begin{aligned}
1 = q_t\, \varepsilon_t^x \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] \\
+ \beta\, \mathbb{E}_t\, q_{t+1} \frac{\lambda_{t+1}}{\lambda_t} \varepsilon_{t+1}^x S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2,
\end{aligned}
\tag{12}
$$

and

$$
q_t = \beta\, \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left[q_{t+1}(1-\delta) + z_{t+1} R_{t+1}^k - \Psi(z_{t+1})\right]. \tag{13}
$$

Equation (12) can be interpreted as an investment Euler equation which describes the optimal investment trajectory. Equation (13) stipulates that q_t is equal to the expected discounted stream of future capital return, corrected for the utilization and depreciation rates.

Finally, Equations (8) and (9) yield a modified UIP condition that takes into account the country specific risk:

$$
R_t^n = \Omega(FA_t, \epsilon_t^{\text{risk}}) R_t^{n,*}. \tag{14}
$$

Aggregate consumption is assumed to be given by a CES index of domestically produced and imported goods according to:

$$
C_t = \left[(1-\alpha_c)^{\frac{1}{\eta_c}} (C_t^d)^{\frac{\eta_c - 1}{\eta_c}} + \alpha_c^{\frac{1}{\eta_c}} (C_t^m)^{\frac{\eta_c - 1}{\eta_c}}\right]^{\frac{\eta_c}{\eta_c - 1}},
$$

where C_t^d and C_t^m denote real consumption of domestic and imported goods respectively, α_c is the share of imports in consumption and η_c is the elasticity of substitution between domestically produced and imported consumption goods in Estonia.

Households maximize C_t subject to the following two expenditure constraints:

$$
P_t^d C_t^d + P_t^m C_t^m = P_t^c\, C_t,
$$

where the consumer price index P_t^c for the Estonian economy is given by:[15]

$$
P_t^c = \left[(1-\alpha_c)(P_t^d)^{1-\eta_c} + \alpha_c (P_t^m)^{1-\eta_c}\right]^{\frac{1}{1-\eta_c}}. \tag{15}
$$

[14] Note that when there is no investment adjustment cost, i.e. when $S\left(\frac{I_t}{I_{t-1}}\right) = 0$, the investment dynamics equation implies that $q_t = \frac{1}{\varepsilon_t^x}$, that is the Tobin's Q is equal to the replacement cost of capital (the relative cost of capital). Furthermore, if $\varepsilon_t^x = 1$ for all $t \geq 0$, as in the standard neoclassical growth model, then $q_t = 1$.

[15] The log-linear version of this equation is:

$$
\widehat{\pi}_t^c = (1-\alpha_c)(\gamma^c)^{1-\eta_c}\, \widehat{\pi}_t^d + \alpha_c\, (\gamma^m)^{1-\eta_c}\, \widehat{\pi}_t^m,
$$

where γ^c and γ^m are the steady state relative prices defined in Appendix 8.1.

In this expression P_t^d denotes the domestic price index, while P_t^m stands for the imported consumption good price index; both are expressed in the domestic currency. From this maximization exercise the following two demand functions are obtained:

$$C_t^d = (1-\alpha_c)\left(\frac{P_t^d}{P_t^c}\right)^{-\eta_c} C_t\,, \tag{16}$$

and

$$C_t^m = \alpha_c\left(\frac{P_t^m}{P_t^c}\right)^{-\eta_c} C_t\,. \tag{17}$$

3.1.1. Labour Supply

Each household is a monopolistic supplier of a differentiated labour service required by the domestic intermediate good producers.[16] The households can therefore set their own wages subject to the substitutability between different labour services determined by the time-varying parameter λ_t^w. After setting its wage, each household inelastically supplies the required labour effort, measured in working hours, at this wage rate.

The analytical framework that leads to an equation describing dynamics of the real wage in the economy is similar to the one used to derive the aggregate price dynamics in the next section. A labour aggregator hires differentiated labour services from the households and transforms them into the homogenous production factor L_t using the following technology:

$$L_t = \left[\int_0^1 (L_t^i)^{\frac{1}{1+\lambda_t^w}}\, di\right]^{1+\lambda_t^w}, \tag{18}$$

where L_t^i denotes i-th household's labour effort, L_t is the aggregated labour supply, and λ_t^w is a stationary wage mark-up shock given by $\widehat{\lambda}_t^w - \lambda^w = \rho_w(\widehat{\lambda}_{t-1}^w - \lambda^w) + u_t^w$, $u_t^w \sim \mathrm{WN}(0,\sigma_w^2)$, where λ_w is the steady-state wage mark-up parameter.[17]

The maximization problem faced by the labour aggregator is following:

$$\max_{\{L_t^i\,:\,i\in[0,1]\}} W_t L_t - \int_0^1 W_t^i L_t^i\, di \qquad \text{subject to Equation (18),}$$

[16] The main references are Kollmann (2001), Erceg et al. (2000), and Christiano, Eichenbaum and Evans (2005). The most recent references are Adolfson et al. (2007a) and Fernandez-Villaverde and Rubio-Ramirez (2007). The latter has a good mathematical appendix with detailed derivations of all relevant formulas.

[17] See Chari et al. (2008) for a discussion and criticism of the wage markup shock and other structural shocks in New Keynesian DSGE models.

from where the demand for i-th household's labour effort L_t^i is given by:

$$L_t^i = \left(\frac{W_t^i}{W_t}\right)^{-\frac{1+\lambda_t^w}{\lambda_t^w}} L_t \quad \forall i \in [0,1], \quad \text{where:} \quad W_t = \Big[\int_0^1 (W_t^i)^{-\frac{1}{\lambda_t^w}} di\Big]^{-\lambda_t^w}. \tag{19}$$

It is also assumed that not all households can optimally re-adjust their wages in each time period. Using Calvo (1983) framework, a fraction $1-\theta_w$ of all households can optimally set their wages in each time period. The remaining households are assumed to index their wages to the past inflation according to the following formula:

$$W_{t+1}^i = (\pi_t^c)^{\tau_w} W_t^i. \tag{20}$$

Faced with these constraints, households set their wages optimally, taking into account the probability of being unable to re-adjust them for a number of time periods into the future. Each household solves the following maximization problem, which is a part of the Lagrangian equation (3), subject to the labour demand function (19) and the wage indexation formula (20):

$$\max_{W_t^i} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \Big[-\frac{\varepsilon_{t+k}^{\beta}}{1+\sigma_l} (L_{t+k}^i)^{1+\sigma_l} + \lambda_{t+k} \prod_{s=1}^{k} \frac{(\pi_{t+s-1}^c)^{\tau_w}}{\pi_{t+s}^c} \frac{W_t^i}{P_t^c} L_{t+k}^i \Big]$$

s.t.

$$L_{t+k}^i = \left[\prod_{s=1}^{k} (\pi_{t+s-1}^c)^{\tau_w} \frac{W_t^i}{W_{t+k}}\right]^{-\frac{1+\lambda_{t+k}^w}{\lambda_{t+k}^w}} L_{t+k} \quad \text{for each } k \geq 0.$$

The first-order condition from this maximization problem needs to be combined with the aggregate wage index law of motion:

$$W_t^{-\frac{1}{\lambda_t^w}} = \theta_w \big[W_{t-1}(\pi_{t-1}^c)^{\tau_w}\big]^{-\frac{1}{\lambda_t^w}} + (1-\theta_w)(\bar{W}_t)^{-\frac{1}{\lambda_t^w}},$$

where $\bar{W}_t$ denotes the optimal wage set by the households in time period t.

The resulting real wage dynamics in log-linear form is given by:

$$\begin{aligned} \widehat{w}_t =& \frac{\beta}{1+\beta} \mathbb{E}_t \widehat{w}_{t+1} + \frac{1}{1+\beta} \widehat{w}_{t-1} + \frac{\beta}{1+\beta} \mathbb{E}_t \widehat{\pi}_{t+1}^c - \frac{1+\beta\tau_w}{1+\beta} \widehat{\pi}_t^c + \frac{\tau_w}{1+\beta} \widehat{\pi}_{t-1}^c \\ & - \frac{1}{1+\beta} \frac{(1-\beta\theta_w)(1-\theta_w)}{\big(1+\frac{1+\lambda_w}{\lambda_w}\sigma_l\big)\theta_w} \big[\widehat{w}_t - \sigma_l \widehat{l}_t - \frac{\sigma_c}{1-h}(\widehat{c}_t - h\widehat{c}_{t-1})\big] + \widehat{\lambda}_t^w. \end{aligned} \tag{21}$$

When wages are completely flexible, that is when $\theta_w = 0$, the real wage dynamics is described by (10).

3.2. Firms

3.2.1. Final Good Producers

Final good is produced using the following aggregation technology, where the intermediate goods Y_t^j are indexed by $j \in [0,1]$:[18]

$$Y_t = \Big[\int_0^1 (Y_t^j)^{\frac{1}{1+\lambda_t^p}} \, dj \Big]^{1+\lambda_t^p} ,$$

where $\log \lambda_t^p - \lambda^p = \rho_p (\log \lambda_{t-1}^p - \lambda^p) + u_t^p$, $u_t^p \sim \mathrm{WN}(0, \sigma_p^2)$ is a stationary price mark-up shock, and λ^p is the steady-state mark-up parameter.[19] λ_t^p is interpreted as a cost push shock in the inflation equation.

The cost minimization condition[20] in the final good sector can be written in the form of a demand function for the intermediate good Y_t^j:

$$Y_t^j = \left(\frac{P_t^j}{P_t^d} \right)^{-\frac{1+\lambda_t^p}{\lambda_t^p}} Y_t \,, \quad \forall j \in [0,1] \,, \tag{22}$$

where P_t^j is the price of the intermediate good j and P_t^d is the domestic price index, which can be written as:

$$P_t^d = \Big[\int_0^1 (P_t^j)^{-\frac{1}{\lambda_t^p}} \, dj \Big]^{-\lambda_t^p} .$$

3.2.2. Intermediate Good Producers

Firms producing intermediate goods operate in a monopolistically competitive market. They hire labour and capital from households, paying the salary

[18] In a standard set up, the aggregation technology is given by:

$$Y_t = \Big[\int_0^1 (Y_t^j)^{\frac{\varepsilon-1}{\varepsilon}} \, dj \Big]^{\frac{\varepsilon}{\varepsilon-1}} ,$$

where ε is the price elasticity of demand for good j. It is known that the gross markup $(1+\lambda^p)$ is equal to $\frac{\varepsilon}{\varepsilon-1}$. In the paper ε is substituted with the expression in terms of the markup with an additional assumptions that it is time varying $\varepsilon_t = \frac{1+\lambda_t^p}{\lambda_t^p}$.

[19] Note that λ^p does not enter into any of the log-linearized equations. It is relevant only for the steady state calculations, refer to Appendix 8.4.

[20] This condition is obtained by solving the following cost minimization problem:

$$\min_{\{Y_t^j \,:\, j\in[0,1]\}} \int_0^1 P_t^j Y_t^j dj \quad \text{s.t.} \quad \Big[\int_0^1 (Y_t^j)^{\frac{1}{1+\lambda_t^p}} \, dj \Big]^{1+\lambda_t^p} \geq Y_t \,.$$

W_t and capital return R_t^k. Each firm, indexed by $j \in [0,1]$, produces Y_t^j units of differentiated output using the following Cobb-Douglas production technology:

$$Y_t^j = A_t\big(\widetilde{K}_{t-1}^j\big)^{\alpha}\big(L_t^j\big)^{1-\alpha} - \Phi^j\,, \tag{23}$$

where $\widetilde{K}_{t-1}^j$ is the effective capital stock given by $\widetilde{K}_{t-1}^j = z_t K_{t-1}^j$, Φ^j is the fixed cost term needed to ensure zero profit in the steady state (see Appendix 8.4), and $\log A_t = \rho_a \log A_{t-1} + u_t^a$, $u_t^a \sim \mathrm{WN}(0, \sigma_a^2)$ is a stationary technology shock common for all firms.

Firms minimize costs subject to the technology constraint. Their objective function is:

$$\min_{\{\widetilde{K}_{t-1}^j, L_t^j\}} \frac{W_t}{P_t^d} L_t^j + R_t^k \widetilde{K}_{t-1}^j \quad \text{s.t.} \quad \bar{Y}_t^j = A_t\big(\widetilde{K}_{t-1}^j\big)^{\alpha}\big(L_t^j\big)^{1-\alpha}.$$

The associated Lagrangian is given by:

$$\min_{\{\widetilde{K}_{t-1}^j, L_t^j\}} \frac{W_t}{P_t^d} L_t^j + R_t^k \widetilde{K}_{t-1}^j + \zeta_t\Big[\bar{Y}_t^j - A_t\big(\widetilde{K}_{t-1}^j\big)^{\alpha}\big(L_t^j\big)^{1-\alpha}\Big],$$

from where the first-order conditions are:

$$(\partial \widetilde{K}_{t-1}^j) \quad R_t^k - \zeta_t\, A_t\, \alpha\big(\widetilde{K}_{t-1}^j\big)^{\alpha-1}\big(L_t^j\big)^{1-\alpha} = 0\,, \tag{24}$$

$$(\partial L_t^j) \quad \frac{W_t}{P_t^d} - \zeta_t\, A_t\,(1-\alpha)\big(\widetilde{K}_{t-1}^j\big)^{\alpha}\big(L_t^j\big)^{-\alpha} = 0\,, \tag{25}$$

where the Lagrange multiplier ζ_t represents the real marginal cost.

Solving (25) for the Lagrange multiplier and substituting the result into (24) gives the optimal capital-labour ratio:

$$\frac{\widetilde{K}_{t-1}^j}{L_t^j} = \frac{\alpha}{1-\alpha}\,\frac{W_t}{P_t^d}\,\frac{1}{R_t^k}\,. \tag{26}$$

Using this result to substitute out $\frac{\widetilde{K}_{t-1}^j}{L_t^j}$ in Equation (25), an expression for the real marginal cost obtains:

$$MC_t = \frac{1}{A_t}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{W_t}{P_t^d}\right)^{1-\alpha}(R_t^k)^{\alpha}\,. \tag{27}$$

Intermediate good producers also face another type of problem. Each period, only a fraction $1-\theta_p$ of them, randomly chosen, can optimally re-adjust their prices (see Calvo, 1983). For those that cannot re-optimize, prices are indexed to past inflation as follows:

$$P^j_{t+1} = (\pi^d_t)^{\tau_p} P^j_t \,,$$

where $\pi^d_t := \frac{P^d_t}{P^d_{t-1}}$ is the gross rate of domestic inflation, and τ_p is the parameter governing the degree of price indexation.

In each time period $t \geq 0$, intermediate good producers maximize the stream of expected discounted profits:[21]

$$\max_{P^j_t} \; \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_p)^k \, \frac{\lambda_{t+k}}{\lambda_t} \Big[\prod_{s=1}^{k} (\pi^d_{t+s-1})^{\tau_p} \frac{P^j_t}{P^d_{t+k}} - MC_{t+k} \Big] Y^j_{t+k} \,,$$

subject to the sequence of intermediate good demand functions by the final good producers; see (22):

$$Y^j_{t+k} = \Big[\prod_{s=1}^{k} (\pi^d_{t+s-1})^{\tau_p} \frac{P^j_t}{P^d_{t+k}} \Big]^{-\frac{1+\lambda^p_{t+k}}{\lambda^p_{t+k}}} Y_{t+k} \quad \text{for each } k \geq 0 \,.$$

The first order condition for this maximization problem, written in terms of the optimal price $\bar{P}_t$, is following:[22]

$$\begin{aligned} &\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_p)^k \lambda_{t+k} \, \frac{1}{\lambda^p_{t+k}} \Big[\prod_{s=1}^{k} \frac{(\pi^d_{t+s-1})^{\tau_p}}{\pi^d_{t+s}} \Big]^{-\frac{1}{\lambda^p_{t+k}}} \frac{\bar{P}_t}{P^d_t} \, Y_{t+k} \\ &\qquad = \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_p)^k \, \lambda_{t+k} \, \frac{1+\lambda^p_{t+k}}{\lambda^p_{t+k}} \Big[\prod_{s=1}^{k} \frac{(\pi^d_{t+s-1})^{\tau_p}}{\pi^d_{t+s}} \Big]^{-\frac{1+\lambda^p_{t+k}}{\lambda^p_{t+k}}} MC_{t+k} \, Y_{t+k} \,. \end{aligned} \tag{28}$$

Given that in each time period a fraction of firms can re-adjust their prices optimally, while the rest index their prices using the previous period's inflation rate, the aggregate price index evolves according to the following weighted average formula:

$$(P^d_t)^{-\frac{1}{\lambda^p_t}} = \theta_p \big[(\pi^d_{t-1})^{\tau_p} P^d_{t-1} \big]^{-\frac{1}{\lambda^p_t}} + (1-\theta_p)(\bar{P}_t)^{-\frac{1}{\lambda^p_t}} \,. \tag{29}$$

[21] Detailed derivation of the New Keynesian Phillips curve equation (30) is not reported in this paper. It can be found in Walsh (2003), Adolfson et al. (2007a), and Fernandez-Villaverde (2007), among others.

[22] Since all firms face the same technology shock and the resulting optimal capital-output ratio is similar across all intermediate producers, the optimal price $\bar{P}_t$ is the same for all firms. Solving this equation for $\bar{P}_t$ and assuming flexible prices ($\theta_p = 0$) leads to the standard monopolistic competition condition whereby each firm sets its price as a markup over the nominal marginal cost $\bar{P}_t = (1+\lambda^p_t) MC_t$.

Log-linearizing and solving the system of equations given by (29) and the first-order condition for the optimal price (28) leads to an equation describing dynamics of the domestic inflation rate. It is given by the hybrid New Keynesian Phillips Curve:

$$\widehat{\pi}_t^d = \frac{\beta}{1+\beta\tau_p}\,\mathbb{E}_t\,\widehat{\pi}_{t+1}^d + \frac{\tau_p}{1+\beta\tau_p}\widehat{\pi}_{t-1}^d + \frac{1}{1+\beta\tau_p}\frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p}\widehat{mc}_t + \widehat{\lambda}_t^p\,. \tag{30}$$

Note that when prices are fully flexible, i.e. $\theta_p = 0$, and the price mark-up shock is zero, Equation (30) reduces to the usual flexible price condition where the real marginal cost is equal to one.

3.3. Importers

The import and export sectors in the EP DSGE model are based on Adolfson et al. (2007a).[23] The import sector consists of a large number of firms that buy a homogenous good in the euro area market and turn it into differentiated consumption goods using a brand naming technology; i.e. without costs. These differentiated consumption goods are then sold to domestic households subject to price stickiness in the local currency.

Importing firms buy the euro area homogenous consumption good at price P_t^*, which is the consumer price index of the euro area. The framework in which these importing firms operate is identical to the one of the intermediate good producers in terms of price setting behavior. Thus, in each time period, only a fraction of importers $1-\theta_m$ is allowed to optimally set their prices. The remaining fraction θ_m of importers adjusts their prices according to the indexation formula:

$$P_{t+1}^{j,m} = (\pi_t^m)^{\tau_m} P_t^{j,m}\,,$$

where the following definition are used: $\pi_t^m := \frac{P_t^m}{P_{t-1}^m}$ is the gross import price inflation rate, $P_t^m = \left[\int_0^1 (P_t^{j,m})^{-1/\lambda_t^m} dj\right]^{-\lambda_t^m}$ is the import price index, $\widehat{\lambda}_t^m = \rho_m\,\widehat{\lambda}_{t-1}^m + u_t^m$, $u_t^m \sim \text{WN}(0,\sigma_m^2)$ is a stationary price mark-up shock on imports, and τ_m is the import price indexation coefficient. The final imported

[23] In Adolfson et al. (2007a) there is a distinction between imported consumption and investment goods. This version of the EP DSGE model does not make this distinction because the statistical data related to the prices of imported investment goods for Estonia is not readily available. This would make estimation of the corresponding Phillips Curve parameters difficult. It is therefore assumed that only consumption goods and services are imported. The same applies to the export sector in Subsection 3.4.

good is a composite of the continuum of $j \in [0,1]$ differentiated imported goods, each supplied by a different firm and priced at $P_t^{j,m}$, which is described by the CES aggregator:

$$C_t^m = \left[\int_0^1 (C_t^{j,m})^{\frac{1}{1+\lambda_t^m}} dj \right]^{1+\lambda_t^m} .$$

The previous equation implies that the demand function faced by an individual importing firms is given by:

$$C_t^{j,m} = \left(\frac{P_t^{j,m}}{P_t^m} \right)^{-\frac{1+\lambda_t^m}{\lambda_t^m}} C_t^m .$$

Importing firms maximize their profits subject to the Calvo (1983) price stickiness restriction.[24] The resulting import price inflation dynamics is given by the following equation in log-linear form:

$$\widehat{\pi}_t^m = \frac{\beta}{1+\beta\tau_m} \mathbb{E}_t \,\widehat{\pi}_{t+1}^m + \frac{\tau_m}{1+\beta\tau_m} \widehat{\pi}_{t-1}^m + \frac{1}{1+\beta\tau_m} \frac{(1-\theta_m)(1-\beta\theta_m)}{\theta_m} \widehat{mc}_t^m + \widehat{\lambda}_t^m . \tag{31}$$

3.4. Exporters

Exporters buy the final good from the domestic market and differentiate it by brand naming. They sell the continuum of differentiated consumption

[24]Each importer $j \in [0,1]$ is assumed to set $P_t^{j,m}$ in order to maximize the discounted stream of future profits:

$$\max_{P_t^{j,m}} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_m)^k \frac{\lambda_{t+k}}{\lambda_t} \Big[\prod_{s=1}^{k} (\pi_{t+s-1}^m)^{\tau_m} \frac{P_t^{j,m}}{P_{t+k}^m} - MC_{t+k}^m \Big] C_{t+k}^{j,m}$$

s.t.

$$C_{t+k}^{j,m} = \left[\prod_{s=1}^{k} (\pi_{t+s-1}^m)^{\tau_m} \frac{P_t^{j,m}}{P_{t+k}^m} \right]^{-\frac{1+\lambda_{t+k}^m}{\lambda_{t+k}^m}} C_{t+k}^m \quad \text{for each } k \geq 0 ,$$

where $MC_{t+k}^m = \frac{\bar{e}\, P_{t+k}^*}{P_{t+k}^m}$ for $k \geq 0$. The first-order condition derived from this maximization problem needs to be combined with the aggregate import price law of motion:

$$(P_t^m)^{-\frac{1}{\lambda_t^m}} = \theta_m \big[(\pi_{t-1}^m)^{\tau_m} P_{t-1}^m \big]^{-\frac{1}{\lambda_t^m}} + (1-\theta_m)(\bar{P}_t^m)^{-\frac{1}{\lambda_t^m}} .$$

After solving and log-linearizing this system of equations, the final expression (31) is obtained.

goods to households in the euro area. The nominal marginal cost is therefore equal to the price of the domestic good P_t^d. Since only consumption goods are exported, each exporting firm $j \in [0,1]$ faces the following demand function for its product in each time period $t \geq 0$:

$$C_t^{j,e} = \left(\frac{P_t^{j,e}}{P_t^e} \right)^{-\frac{1+\lambda_t^e}{\lambda_t^e}} X_t \,,$$

where X_t denotes aggregate export, P_t^e is the export price index expressed in the local currency of the export market, and $\log \lambda_t^e = \log \lambda^e + u_t^e$, $u_t^e \sim \mathrm{WN}(0, \sigma_{\lambda^e}^2)$ is the stochastic markup on differentiated export goods. Once again, the price stickiness faced by exporting firms implies that a fraction $1 - \theta_e$ of them can re-adjust prices in each period. For the rest, prices evolve according to the indexation formula:

$$P_{t+1}^{j,e} = (\pi_t^e)^{\tau_e} P_t^e \,.$$

Exporters maximize their profits subject to the price stickiness restriction.[25] The resulting export price inflation dynamics is given by the following equation in log-linear form:

$$\widehat{\pi}_t^e = \frac{\beta}{1+\beta\tau_e} \mathbb{E}_t \, \widehat{\pi}_{t+1}^e + \frac{\tau_e}{1+\beta\tau_e} \widehat{\pi}_{t-1}^e + \frac{1}{1+\beta\tau_e} \frac{(1-\theta_e)(1-\beta\theta_e)}{\theta_e} \widehat{mc}_t^e + u_t^e \,. \tag{32}$$

In addition, EP DSGE model assumes that the Estonian economy is small relative to the euro area economy, and hence plays just a negligible part in

[25]Each exporter $j \in [0,1]$ is assumed to set $P_t^{j,e}$ in order to maximize the discounted stream of future profits:

$$\max_{P_t^{j,e}} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_e)^i \frac{\lambda_{t+k}}{\lambda_t} \Big[\prod_{s=1}^{k} (\pi_{t+s-1}^e)^{\tau_e} \frac{P_t^{j,e}}{P_{t+k}^e} - MC_{t+k}^e \Big] C_{t+k}^{j,e}$$

s.t.

$$C_{t+k}^{j,e} = \left[\prod_{s=1}^{k} (\pi_{t+s-1}^e)^{\tau_e} \frac{P_t^{j,e}}{P_{t+k}^e} \right]^{-\frac{1+\lambda_{t+k}^e}{\lambda_{t+k}^e}} X_{t+k} \quad \text{for each } k \geq 0 \,,$$

where $MC_{t+k}^e = \frac{P_{t+k}^d}{\bar{e}\, P_{t+k}^e}$ for $k \geq 0$. The first-order condition derived from this maximization problem needs to be combined with the aggregate export price law of motion:

$$(P_t^e)^{-\frac{1}{\lambda_t^e}} = \theta_e \big[(\pi_{t-1}^e)^{\tau_e} P_{t-1}^e \big]^{-\frac{1}{\lambda_t^e}} + (1-\theta_e)(\bar{P}_t^e)^{-\frac{1}{\lambda_t^e}} \,.$$

After solving and log-linearizing this system of equations, the final expression (32) is obtained.

the aggregate euro area consumption. Assuming that aggregate euro area consumption is well approximated by a CES function, demand for Estonian exports is given by:

$$X_t = \left(\frac{P_t^e}{P_t^*}\right)^{-\eta_*} C_t^* ,$$

where η_* denotes elasticity of substitution between imported and domestically produced consumption goods in the euro area, and C_t^* is the euro area aggregate consumption. Furthermore, using a simplifying assumption that $Y_t^* = C_t^*$, the following export demand function is obtained:[26]

$$X_t = \left(\frac{P_t^e}{P_t^*}\right)^{-\eta_*} Y_t^* . \tag{33}$$

3.5. Policies

3.5.1. Fiscal Policy

Fiscal policy is exogenous and assumed to behave as follows:

$$\log G_t - \log G = \rho_g\big(\log G_{t-1} - \log G\big) + u_t^g , \tag{34}$$

where $G := g\,Y$ is the steady-state level of government spending, $g := \frac{G}{Y}$ is the parameter governing the share of government expenditures in the total output, and $u_t^g \sim \text{WN}(0, \sigma_g^2)$ is the government spending shock. In addition, the balanced budget condition implies that $G_t = -T_t$, where $-T_t$ are lump sum taxes payed by the households in the economy. There is no active government tax policy in the model.

3.5.2. Monetary Policy

The monetary policy of Estonia is subject to the currency board arrangement and free capital mobility between the domestic and euro area markets. The UIP condition derived previously in (14) implies that the domestic nominal interest rate is linked to the euro area nominal interest rate via the country specific risk premium function:

$$R_t^n = \Omega(FA_t, \epsilon_t^{\text{risk}}) R_t^{n,*} .$$

In other words, R_t^n is determined by the monetary policy in the euro area, by fluctuations in the net foreign assets FA_t, and by an idiosyncratic shock ϵ_t^{risk}.

[26]This assumption is not fully correct, but does not affect the estimation results reported in Section 6. It is stated in Section 4 that $Y_t^* = C_t^* + I_t^* + G_t^* + \Psi(z_t^*)K_{t-1}^*$, whereby dynamics of Y_t^* and C_t^* is different.

3.6. The Aggregate Resource Constraint

The aggregate resource constraint is:

$$Y_t = C_t^d + C_t^m + I_t + G_t + \Psi(z_t)K_{t-1} + X_t - M_t\,, \tag{35}$$

where imports are defined as:

$$M_t := C_t^m\,. \tag{36}$$

After substituting all the components of domestic output into (35) using Equations (16), (36) and (33) the following expression is obtained:

$$Y_t = (1-\alpha_c)\left(\frac{P_t^d}{P_t^c}\right)^{-\eta_c} C_t + I_t + G_t + \Psi(z_t)K_{t-1} + \left(\frac{P_t^e}{P_t^*}\right)^{-\eta_*} Y_t^*\,.$$

3.7. The Net Foreign Assets

Evolution of the net foreign assets is described by the following equation:

$$\overline{e}\,B_t^* = \overline{e}\,P_t^e X_t - \overline{e}\,P_t^* M_t + \Omega(FA_t, \epsilon_t^{\text{risk}})\,\overline{e}\,R_{t-1}^{n,*} B_{t-1}^*\,. \tag{37}$$

Dividing both sides of this equation by P_t^d and using the definitions of X_t, M_t, MC_t^e and C_t^m from Equations (33), (36), (17), and Footnote 25, Equation (37) can be written as:

$$FA_t = \frac{1}{MC_t^e}\left(\frac{P_t^e}{P_t^*}\right)^{-\eta_*} Y_t^* - \frac{P_t^*}{P_t^d}\,\alpha_c\left(\frac{P_t^m}{P_t^d}\right)^{-\eta_c} C_t + \Omega(FA_t, \epsilon_t^{\text{risk}})\,R_{t-1}^{n,*}\,\frac{FA_{t-1}}{\pi_t^d}\,.$$

4. Key Equations: Euro Area Economy

The euro area part of the EP DSGE model is based on the seminal paper by Smets and Wouters (2003).[27] In contrast to Adolfson et al. (2007a), where the rest of the world is described by a low-dimensional VAR system, this paper adopts a full-fledged DSGE model as a counterpart to the Estonian economy

[27] Initially, the idea was to model the euro area as a basic three-equation NK DSGE model, and estimate it jointly with the Estonian economy part. This is more in line with the spirit of small open economy DSGE models, where the rest of the world is often described by a simple three-equation VAR system. Later, a decision was taken to adopt a much richer framework for characterizing the euro area economy, afforded by the benchmark model in Smets and Wouters (2003). An interesting extension of the present approach would be to consider an open economy model for the euro area as well, allowing to study the impact of the rest of the world on the Estonian economy through the corresponding interaction with the euro area. In this respect, Adolfson et al. (2007a) is an excellent reference.

part described in Section 3. Since one of the main interests of the EP DSGE model is to examine the propagation mechanism through which various euro area structural disturbances impact on the Estonian economy, it is necessary to have a rich euro area model that incorporates a wide range of "deep" shocks with clear economic interpretations attached to them. However, due to the increased dimensions of the euro area part in the EP DSGE model and associated computational costs, all deep parameters related to the euro area part are calibrated according to the values in Smets and Wouters (2003), while the coefficients linked to the structural shocks are estimated jointly with the parameters of the Estonian part of the model.

In terms of equations, the model is similar to the Estonian economy part in Section 3, but with a few substantial differences; see Smets and Wouters (2003) and references therein for a detailed description of the model.[28] The differences are due to the fact that it is a closed economy model having an independent monetary policy described by a monetary policy rule.

The aggregate resource constraint in the euro area part of the EP DSGE model is given by:

$$Y_t^* = C_t^* + I_t^* + G_t^* + \Psi(z_t^*)K_{t-1}^* \,.$$

The monetary policy rule is as follows:

$$\begin{aligned}\widehat{r}_t^{n,*} =& \phi_m \, \widehat{r}_{t-1}^{n,*} + (1-\phi_m)\big[r_\pi \widehat{\pi}_{t-1}^* + r_y(\widehat{y}_{t-1}^* - \widehat{y}_{t-1}^{p,*})\big] \\ &+ r_{\Delta\pi}(\widehat{\pi}_t^* - \widehat{\pi}_{t-1}^*) + r_{\Delta y}\big[\widehat{y}_t^* - \widehat{y}_t^{p,*} - (\widehat{y}_{t-1}^* - \widehat{y}_{t-1}^{p,*})\big] + \widehat{\varepsilon}_t^{r,*} \,,\end{aligned} \tag{38}$$

where $\widehat{\varepsilon}_t^{r,*} = \rho_r^* \, \widehat{\varepsilon}_{t-1}^{r,*} + u_t^{r,*}$, $u_t^{r,*} \sim \mathrm{WN}(0, \sigma_{r,*}^2)$ is the euro area monetary policy shock, and $\widehat{y}_t^{p,*}$ is the logarithm of the potential output level. In DSGE literature, the potential output is defined as the level of output that would prevail under flexible prices and wages in the absence of so-called "inefficient" shocks $u_t^{w,*}, u_t^{p,*}$. As in Smets and Wouters (2003), the model is expanded with the flexible prices and wages version, where $\theta_p^*, \theta_w^*, u_t^{w,*}, u_t^{p,*}$ are set to zero in all time periods, in order to compute the model-consistent output gap $\widehat{y}_t^* - \widehat{y}_t^{p,*}$ used in the monetary policy rule.

[28] All log-linear model equations are reported in Appendix 8.3 as a reference. Note that, differently from the original Smets and Wouters (2003) paper, the inflation objective, price of capital and labour supply shocks are excluded in order to balance the number of shocks and observable variables in the final model. In addition, the relative risk aversion parameter is set to unity.

5. Data and Estimation

5.1. Bayesian Estimation Methodology

The statistical inference for the parameters of the EP DSGE model introduced in Sections 3 and 4 is obtained by Bayesian methods, where the associated empirical results are reported in Section 6. Bayesian statistical methods have recently gained popularity in applied macro-economic modeling. For a recent overview of the main literature and methods of the Bayesian analysis of DSGE models, refer to An and Schorfheide (2007). This subsection gives an overview of the main stages of the Bayesian statistical inference for DSGE models.

In contrast to the traditional approach to statistical estimation and testing known under the banner of "frequentist statistics", where the inference based on repeated sampling plays a pivotal role and a "true" model with an unknown but invariant set of parameters is assumed to exist, Bayesian statistics adopts a view that parameters are just "mental constructs that exist only in the mind of the researcher" (see Poirier, 1995). Bayesian statistics is based on a fusion of priors about the model parameters with the likelihood function based on real-world data, where "the likelihood represents a "window" for viewing the observable world shared by a group of researches who agree to disagree in terms of possibly different prior distributions" (see Poirier, 1995). Bayesian statistics knowingly departs from the assumptions of repeated sampling experiments and an underlying data generating process based on an invariant set of unknown parameters, the two assumptions that are crucial to the traditional frequentist approach. Bayesian approach to statistical estimation and testing is arguably better suited for observational sciences such as economics, while the traditional frequentist paradigm is geared toward experimental sciences.

Bayesian statistics can be characterized as a learning process, where observed data collected in $\mathbf{Y}$ is used to learn about the posterior distribution $f(\boldsymbol{\theta} \,|\, \mathbf{Y})$ of a k-dimensional vector of model parameters $\boldsymbol{\theta}$, given the likelihood function $L(\boldsymbol{\theta}; \mathbf{Y})$ and the prior distribution $f(\boldsymbol{\theta})$. This learning process is based on a version of the Bayes' Theorem:

$$f(\boldsymbol{\theta} \,|\, \mathbf{Y}) = \frac{f(\boldsymbol{\theta})L(\boldsymbol{\theta}; \mathbf{Y})}{f(\mathbf{Y})} \propto f(\boldsymbol{\theta})L(\boldsymbol{\theta}; \mathbf{Y})\,, \tag{39}$$

where $f(\mathbf{Y}) = \int_{\mathbf{R}^k} f(\boldsymbol{\theta})L(\boldsymbol{\theta}; \mathbf{Y})d\boldsymbol{\theta}$ can be treated as a normalizing constant for the purpose of posterior inference. The main object of interest for Bayesian inference is the posterior distribution function $f(\boldsymbol{\theta} \,|\, \mathbf{Y})$, which summarizes the information available in data $\mathbf{Y}$ about the vector of parameters $\boldsymbol{\theta}$. The posterior distribution function may further be combined with a statistical loss function in order to arrive to point and interval inference about $\boldsymbol{\theta}$ as well as other forms of statistical decisions involving the vector of parameters.

It follows from expression (39) that Bayesian statistical inference requires both the likelihood function and the prior distribution. The remaining part of this section provides a general overview of the steps involved in the construction of the likelihood function for a typical DSGE model. This discussion is applicable not only to the EP DSGE model introduced in Sections 3 and 4, but also to other DSGE and real business cycle models found in the literature and estimated in the form of first-order linear or log-linear approximations around the steady state. The issues related to the particular choice of priors for the EP DSGE model are deferred to Section 5.2.

In general, the likelihood function of a typical DSGE model cannot be written in closed form as a function of data $\mathbf{Y}$ and model parameters $\boldsymbol{\theta}$. However, the procedure for evaluating the likelihood function for any given $\mathbf{Y}$ and $\boldsymbol{\theta}$ is well known and involves three stages. They are described below in detail.

The first stage involves writing a theoretical macro-economic model as a system of linear expectational and non-expectational equations, including exogenous stochastic processes. Appendices 8.2 and 8.3 list the corresponding system of log-linear equations for the EP DSGE model. Let $\boldsymbol{x}_t$ denote an $m \times 1$ vector of endogenous model variables, $\boldsymbol{y}_t$ denote $n \times 1$ vector of other endogenous model variables, and $\boldsymbol{z}_t$ be $k \times 1$ vector of exogenous structural shocks. A DSGE model can be written in linearized form as follows:

$$\begin{aligned} \mathbf{0} &= \mathbf{A}\boldsymbol{x}_t + \mathbf{B}\boldsymbol{x}_{t-1} + \mathbf{C}\boldsymbol{y}_t + \mathbf{D}\boldsymbol{z}_t \\ \mathbf{0} &= \mathbb{E}_t\big[\mathbf{F}\boldsymbol{x}_{t+1} + \mathbf{G}\boldsymbol{x}_t + \mathbf{H}\boldsymbol{x}_{t-1} + \mathbf{J}\boldsymbol{y}_{t+1} + \mathbf{K}\boldsymbol{y}_t + \mathbf{L}\boldsymbol{z}_{t+1} + \mathbf{M}\boldsymbol{z}_t\big] \\ \boldsymbol{z}_{t+1} &= \mathbf{N}\boldsymbol{z}_t + \boldsymbol{\epsilon}_{t+1}\,, \end{aligned} \tag{40}$$

where a $k \times 1$ vector $\boldsymbol{\epsilon}_t$ of stochastic innovations s.t. $\mathbb{E}_t\,\boldsymbol{\epsilon}_{t+1} = \mathbf{0}$ and its variance–covariance matrix given by $\boldsymbol{\Sigma}$. The vector of model parameters $\boldsymbol{\theta}$ is mapped into the matrices $\mathbf{A}$ to $\boldsymbol{\Sigma}$ of this system according to the theoretical model.

The linear system of expectational equations (40) is solved in the second stage of the likelihood function evaluation. The method of undetermined coefficients stipulates the following solution of the system (40):

$$\begin{aligned} \boldsymbol{x}_t &= \mathbf{P}\boldsymbol{x}_{t-1} + \mathbf{Q}\boldsymbol{z}_t \\ \boldsymbol{y}_t &= \mathbf{R}\boldsymbol{x}_{t-1} + \mathbf{S}\boldsymbol{z}_t \\ \boldsymbol{z}_{t+1} &= \mathbf{N}\boldsymbol{z}_t + \boldsymbol{\epsilon}_{t+1}\,, \end{aligned} \tag{41}$$

where matrices $\mathbf{P}$ to $\mathbf{S}$ are mappings of matrices $\mathbf{A}$ to $\mathbf{M}$ defined in (40) and therefore are also functions of the vector of model parameters $\boldsymbol{\theta}$. Uhlig (1999) gives a comprehensive overview of the method of undetermined coefficients for linear systems of expectational equations like (40), including conditions on dimensions and the ranks of matrices $\mathbf{A}$ to $\boldsymbol{\Sigma}$ that are necessary to obtain the solution (41). In particular, the stability or instability of the system of linear expectational equations (40) depends on the vector of model parameters $\boldsymbol{\theta}$ through the matrices $\mathbf{A}$ to $\mathbf{M}$ and is reflected by the eigenvalues of $\mathbf{P}$ matrix in (41).

Having obtained the system of stochastic difference equations (41) for endogenous variables $\boldsymbol{x}_t$ and $\boldsymbol{y}_t$ and using the law of motion of exogenous stochastic processes $\boldsymbol{z}_t$, the likelihood function $L(\boldsymbol{\theta};\mathbf{Y})$ of a DSGE model is evaluated using the Kalman filter in the third stage of the procedure. The Kalman filter is needed because the endogenous variables of the model in vectors $\boldsymbol{x}_t$ and $\boldsymbol{y}_t$ usually involve certain quantities for which no observable counterparts can be found in macro-economic statistics. Let $m+k \times 1$ dimensional vector $\tilde{\boldsymbol{x}}_t$ be defined as $\tilde{\boldsymbol{x}}_t^{\mathrm{T}} := (\boldsymbol{x}_{t-1}^{\mathrm{T}}, \boldsymbol{z}_t^{\mathrm{T}})$, and let $\tilde{\boldsymbol{y}}_t$ be a vector of observables for each $1 \le t \le T$. The full data matrix is given by $\mathbf{Y} := (\tilde{\boldsymbol{y}}_1^{\mathrm{T}}, \tilde{\boldsymbol{y}}_2^{\mathrm{T}}, \dots, \tilde{\boldsymbol{y}}_T^{\mathrm{T}})$. A DSGE model can be written in the Kalman filter form using the solution (41) as follows:

$$
\begin{aligned}
\tilde{\boldsymbol{x}}_{t+1} &= \begin{pmatrix} \mathbf{P} & \mathbf{Q} \\ \mathbf{0} & \mathbf{N} \end{pmatrix} \tilde{\boldsymbol{x}}_t + \begin{pmatrix} \mathbf{0} \\ \boldsymbol{\epsilon}_t \end{pmatrix} \\
\tilde{\boldsymbol{y}}_t &= \boldsymbol{\Gamma} \tilde{\boldsymbol{x}}_t \,,
\end{aligned}
\tag{42}
$$

where matrix $\boldsymbol{\Gamma}$ maps a subset of endogenous model variables into the observed data, and may include elements of matrices $\mathbf{R}$ and $\mathbf{S}$ from the system (41). In some cases the measurement equation in (42) may include additional measurement errors when the observed data is deemed to be an imperfect counterpart of the endogenous variables $\tilde{\boldsymbol{x}}_t$.[29] The likelihood function $L(\boldsymbol{\theta};\mathbf{Y})$ is evaluated for any given value of the parameter vector $\boldsymbol{\theta}$ using the standard Kalman filter recursions; refer to Hamilton (1994) a detailed overview of the necessary steps.

It is necessary to note that the mapping of parameters $\boldsymbol{\theta}$ into the likelihood function $L(\boldsymbol{\theta};\mathbf{Y})$ of a typical DSGE model is highly complicated, involving nonlinear transformations at the solution stage (41). This might give rise to identifiability issues which are difficult to deal with because of the lack of developed diagnostic methods. Some of the issues related to identification in DSGE models are discussed in Canova (2008) and Iskrev (2008).

Apart from the likelihood and priors, Bayesian statistical inference based on (39) requires a set of techniques to evaluate the posterior distribution $f(\boldsymbol{\theta} \mid \mathbf{Y})$.

[29] Bayesian estimation of the EP DSGE model reported in Section 6 is carried out without adding the measurement errors into the Kalman filter equations.

Specifically, one is usually interested in at least first few moments of $f(\boldsymbol{\theta}\,|\,\mathbf{Y})$, but the accepted current practice requires reporting of estimated posterior densities for each model parameter in the vector $\boldsymbol{\theta}$. Since for a typical DSGE model it is impossible to characterize $f(\boldsymbol{\theta}\,|\,\mathbf{Y})$ analytically, computationally intensive Monte Carlo sampling methods are needed to generate draws from the posterior distribution.[30]

The Metropolis-Hastings Markov chain Monte Carlo sampling algorithm offers a general and easy-to-implement way to draw random numbers from a probability distributions for which no procedural random number generators are available. Its particularly simple implementation is known as the random walk Metropolis-Hastings algorithm and involves the following four steps:

1. Given the previous draw $\boldsymbol{\theta}_{i-1}$ from $f(\boldsymbol{\theta}\,|\,\mathbf{Y})$, generate a candidate draw as follows:

$$\boldsymbol{\theta}_i^* = \boldsymbol{\theta}_{i-1} + \boldsymbol{v}_i \,, \text{ where } \boldsymbol{v}_i \sim \text{i.i.d. with } \mathbb{E}\,\boldsymbol{v}_i = \mathbf{0}\,,\ \mathbb{E}\,\boldsymbol{v}_i\boldsymbol{v}_i^{\mathrm{T}} = \mathbf{S}\,;$$

2. Compute:

$$\alpha_i := \min\left[1, \frac{f(\boldsymbol{\theta}_i^*\,|\,\mathbf{Y})}{f(\boldsymbol{\theta}_{i-1}\,|\,\mathbf{Y})}\right];$$

3. Assign the new draw $\boldsymbol{\theta}_i$ from $f(\boldsymbol{\theta}\,|\,\mathbf{Y})$ as:

$$\boldsymbol{\theta}_i = \begin{cases} \boldsymbol{\theta}_i^* & \text{if } \alpha_i \geq u_i \\ \boldsymbol{\theta}_{i-1} & \text{if } \alpha_i < u_i \end{cases} \,, \text{ where } \boldsymbol{u}_i \sim \text{i.i.d. Uniform}[0,1]\,;$$

4. Repeat Steps 1 to 3 until enough random draws are generated from the distribution $f(\boldsymbol{\theta}\,|\,\mathbf{Y})$.

Parameter $\mathbf{S}$ of the proposal distribution of $\boldsymbol{v}_i$ in the first step of the algorithm needs to be determined prior to the sampling procedure. Given a set of N draws $\{\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_N\}$ from the posterior distribution $f(\boldsymbol{\theta}\,|\,\mathbf{Y})$ supplied by the Metropolis-Hastings or another sampling method, sample moments, kernel density estimates and other posterior statistics can be computed in the usual fashion.

The practical implementation of the steps associated with evaluation of the likelihood function and drawing from the posterior distribution varies from low-level programming of all necessary steps in one of the mathematical programming languages, such as MATLAB®, to using higher-level packages

[30] A good survey of Monte Carlo methods in Bayesian statistics can be found in Robert and Casella (2004).

specifically written for the analysis and estimation of DSGE models, such as Dynare.[31] The latter is used for the simulation and estimation of the EP DSGE model in this paper.

The posterior distributions of the model parameters and all other associated empirical results, which are reported in Section 6 of this paper, have been obtained using Dynare toolbox for MATLAB® as follows. The algorithm is started with a maximization of the posterior kernel, followed by an evaluation of the Hessian matrix at the posterior kernel maximum, which is then used as an input for the main run of the Metropolis-Hastings algorithm to compute the posterior distributions of the model parameters and other statistics.

5.2. Data and Priors

The EP DSGE model for Estonia introduced in Sections 3 and 4 is expressed in the form of log-deviations from the constant steady state. In other words, the model is not designed to explain long-run trends and seasonal fluctuations in the observed macro-economic variables, but rather is focused on the business cycle frequency features of the main macro-economic aggregates. Empirical data series are therefore required to undergo a certain treatment before being used in the evaluation of the model's likelihood function. This section also discusses the choice of priors and calibrated values for the model's structural parameters.

The likelihood function of the EP DSGE model is evaluated using 16 data series, including 6 series that describe the most important euro area macro-economic indicators. The data series used for model estimation are shown in Figure 2. All empirical variables are quarterly, covering the time interval from 1995:2 to 2008:3, thus giving 54 observations per data series. All domestic economy observables are sourced from the EMMA quarterly model of the Estonian economy (refer to Kattai, 2005). Euro area empirical data series are taken from the AWM database (refer to Fagan et al., 2005).

As mentioned previously, since the theoretical model is not designed to pick up seasonal fluctuations in the macro-economic data, all seasonal features of the series are removed prior to the estimation using a filter based on quarterly dummies. The filter allows for a smooth variation in the seasonal pattern throughout the time period spanned by the data.

Individual data series used to evaluate the EP DSGE model's likelihood function are described below:

- Data series for $\widehat{y}_t$ is computed using linearly de-trended real per capita

[31] See Dynare's homepage at `www.cepremap.cnrs.fr/dynare` and Juillard (2004).

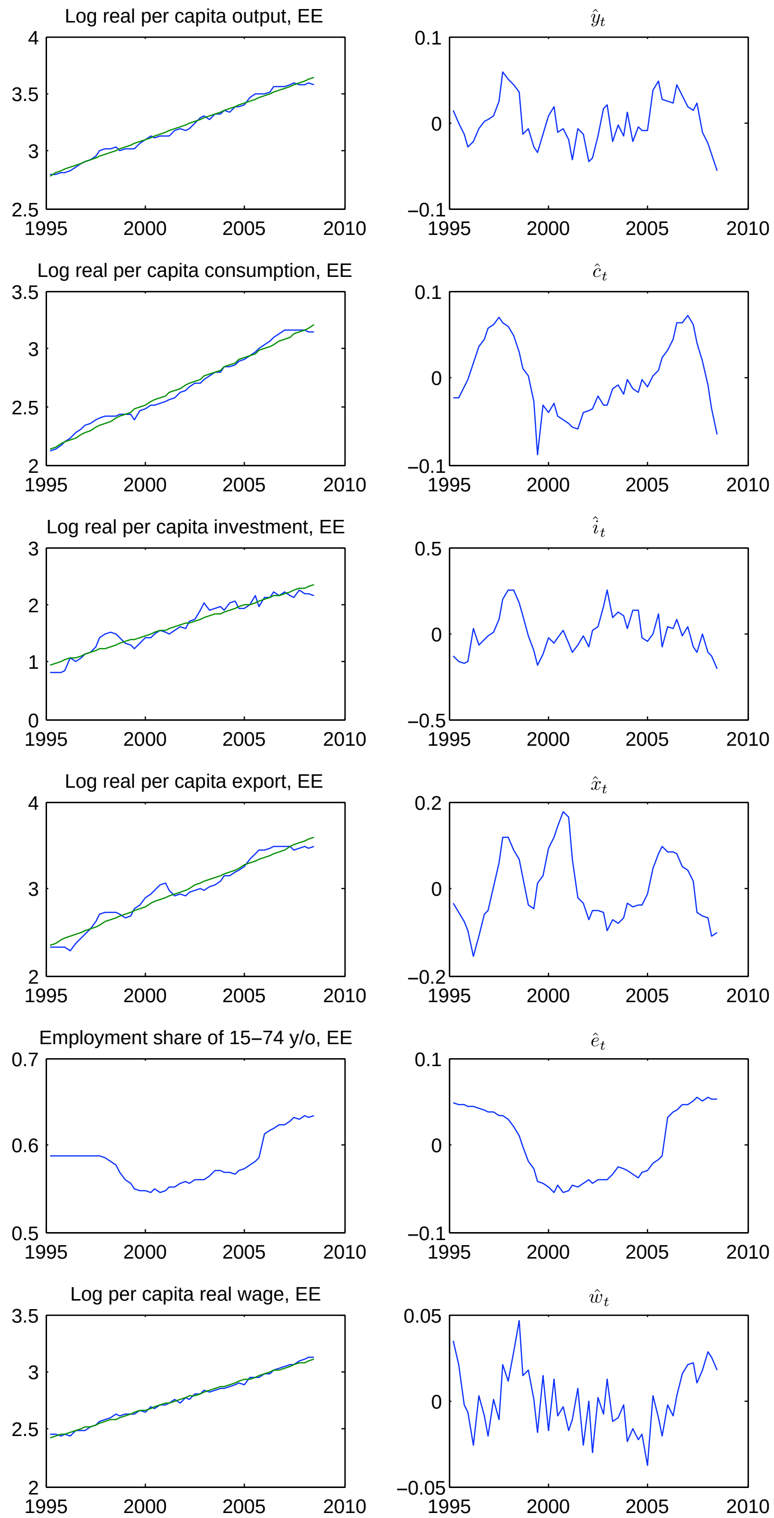

Figure 2: Original log data series with trends (left-hand side) and final observables (right-hand side), 1995:2 to 2008:3

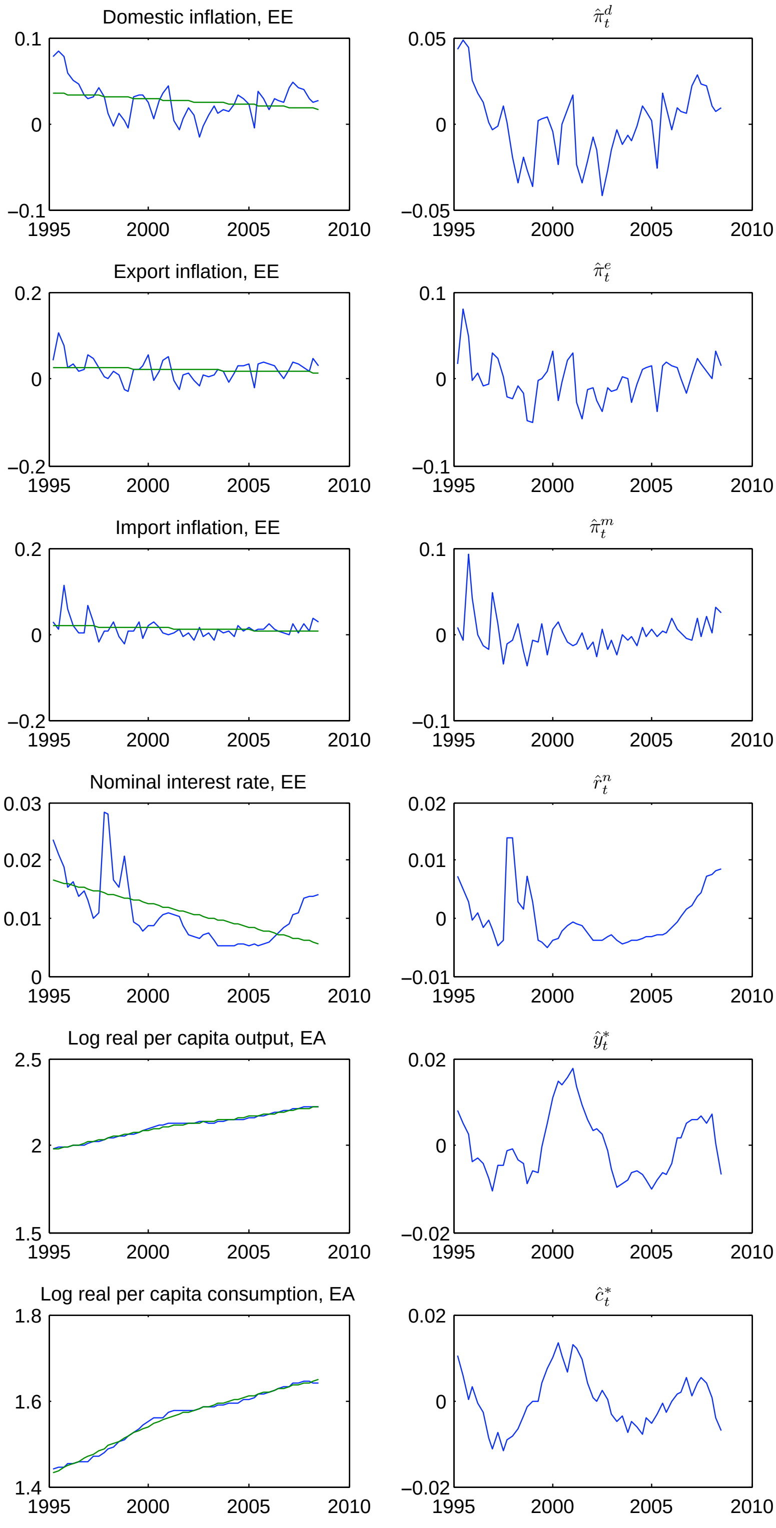

Figure 2: Original log data series with trends (left-hand side) and final observables (right-hand side), 1995:2 to 2008:3 (cont.)

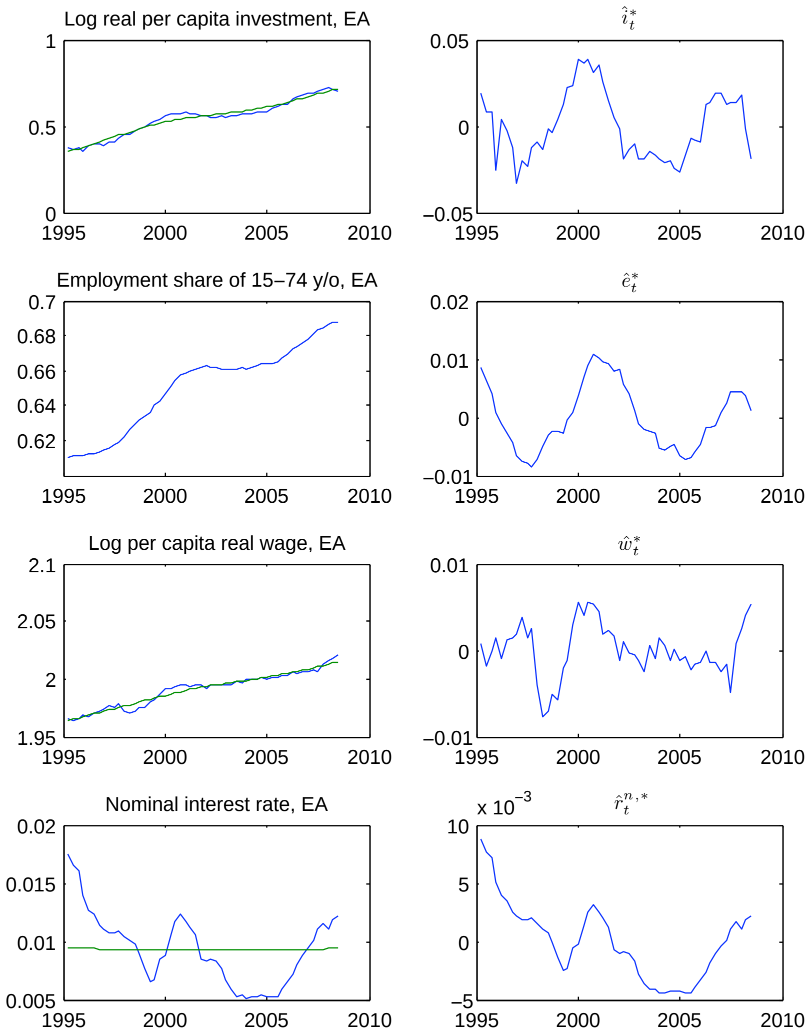

Figure 2: Original log data series with trends (left-hand side) and final observables (right-hand side), 1995:2 to 2008:3 (cont.)

output variable, which itself consists of the sum of real per capita private consumption, real per capita investment, real per capita government consumption and real per capita trade balance;[32] see Figure 2;

- Data series for $\widehat{c}_t$ is based on the national accounts linearly de-trended real per capita private consumption variable; see Figure 2;

- Data series for $\widehat{i}_t$ is the linearly de-trended real per capita firms capital formation series, based on the national accounts statistics; see Figure 2;

- Data series for $\widehat{x}_t$ is calculated as the linearly de-trended real per capita export of goods and services, based on the national accounts statistics; see Figure 2;

- Data series for $\widehat{e}_t$ is computed as the linearly de-trended share of employed in the working age population, where the latter is defined as 15 to 74 year old;[33] see Figure 2;

- Data series for $\widehat{w}_t$ is based on the linearly de-trended per capita real wage, which is calculated as the nominal quarterly wage bill net of social security contributions deflated by the GDP deflator; see Figure 2;

- Data series for $\widehat{\pi}_t^d$, $\widehat{\pi}_t^x$ and $\widehat{\pi}_t^m$ are calculated as linearly de-trended quarter-on-quarler changes in the GDP deflator, export deflator, and import deflator series respectively; see Figure 2;

- Data series for $\widehat{r}_t^n$ is obtained by linearly de-trending the 3-month average deposit rate in Estonia, since no statistics on short-term interest rates on government obligations is available; see Figure 2;

- Data series for the euro area variables $\widehat{y}_t^*$, $\widehat{c}_t^*$, $\widehat{i}_t^*$, $\widehat{e}_t^*$, $\widehat{w}_t^*$ and $\widehat{r}_t^{n,*}$ are Hodrick-Prescott filtered series of the real per capita output, real per capita private consumption, real per capita investment, share of employed in the working age population, real compensation per employee and the nominal 3-month interest rate respectively; see Figure 2.

Similarly to Smets and Wouters (2003), the lack of suitable working hours statistics is mitigated by assuming the following *ad hoc* linkage between the

[32] Definition of the real output is consistent with the national accounts real GDP aggregate, apart from the final consumption of non-profit entities, which is excluded from $\widehat{y}_t$.

[33] Although some corrections for part-time employment are made, there is no reliable statistics on the actual working hours available from Statistics Estonia.

observable economy-wide employment share series $\widehat{e}_t$ and the latent labour effort state variable $\widehat{l}_t$:

$$\widehat{e}_t = \frac{\beta}{1+\beta}\,\mathbb{E}\,\widehat{e}_{t+1} + \frac{1}{1+\beta}\,\widehat{e}_{t-1} + \frac{1}{1+\beta}\,\frac{(1-\beta\chi)(1-\chi)}{\chi}(\widehat{l}_t - \widehat{e}_t)\,,$$

where χ can be interpreted as an employment adjustment parameter that determines how fast the firms are able to bring in or shed the number of workers in the face of fluctuations in the required labour effort. A similar function in terms of the parameters β^* and χ^* is assumed to link $\widehat{e}^*_t$ and $\widehat{l}^*_t$ variables in the euro area part of the model.

It can be observed that the volatility of Estonian macroeconomic data series are three to five times larger than the corresponding euro area one; refer to Figure 2. This is not surprising, considering small size and high degree of openness of the Estonian economy. While none of the "deep" model parameters are directly linked to the volatility of endogenous variables, it is nevertheless reasonable to expect some of the estimation results, aside from the standard errors of the structural shocks, to be affected by this difference.

Table 1: Steady-state shares and calibration

Consumption–income share ($\frac{C}{Y}$)	0.5500
Capital–output ratio (α)	0.4600
Capital depreciation rate (δ)	0.0250
Intertemporal discount factor (β)	0.9875
Steady-state wage mark up (λ^w)	3.0000
Steady-state relative export price (γ^e)	1.0000
Share of imports in consumption (α_c)	0.5000

Notes: The remaining steady-state shares are functions of these and other model parameters; refer to Appendix 8.4. for derivation details.

Regarding the choice of prior distributions and calibrated parameters, this paper follows the usual conventions in the DSGE modeling literature. The prior distributions and associated hyper-parameters for the Estonian part of the EP DSGE model are selected according to Adolfson et al. (2007a); refer to Table 2 for details.[34] In addition, some steady-state shares and few "deep"

[34]It is in the spirit of Bayesian statistics to select priors with a reference to results previously reported in the context of similar studies. The only related empirical DSGE model based on Estonian data is Colantoni (2007), which has many similarities to the EP DSGE design. However, the empirical part of Colantoni (2007) model is relatively unpolished to be considered a good source of priors for the EP DSGE model. In particular, the likelihood function in Colantoni (2007) uses only three observables, while the size of his model in terms of the number of endogenous variables and structural shocks is comparable to the EP DSGE.

parameters of the model, for which good theoretical reference values are available, are calibrated; refer to Table 1. Among the calibrated parameters in the table, the steady-state shares are set to match the corresponding sample averages, capital-output ratio α is taken from Ratto et al. (2008), the intertemporal discount factor β is taken from Lendvai and Roeger (2008), corresponding to the steady-state annual interest rate of 5%, the steady-state wage mark-up parameter λ^w is taken from Smets and Wouters (2003), and the remaining parameters are selected to have empirically plausible steady-state import-to-output ratio.

Recall that the euro area part of the model is estimated partly: priors for the standard error and autoregressive parameters of six structural shocks and the exogenous fiscal policy process are taken from Smets and Wouters (2003). All other parameters of the euro area part of the EP DSGE model are calibrated using the corresponding posterior mode values in Smets and Wouters (2003). Effectively, degenerate priors have been imposed on "deep" parameters of the euro area part of the model, which is needed for computational feasibility reasons.

6. Empirical Results

6.1. Estimated Posterior Distributions

As mentioned in Section 5, posterior distributions and related posterior statistics for the parameters of the EP DSGE model are computed by Metropolis-Hastings sampling algorithm implemented in Dynare toolbox for MATLAB®. Figure 3 shows three diagnostic graphs that are used to assess convergence of the Metropolis–Hastings sampling algorithm to a stationary sampling distribution (refer to Brooks and Gelman, 1998). The figure indicates that an overall convergence is reached after about 2×10^5 draws. Results for the individual parameters are also satisfactory, although some diagnostic measures do suggest instability, especially the ones based on the third moment. However, for all parameters the Metropolis–Hastings sampling algorithm convergences after about 2×10^5 draws.[35]

Posterior density graphs of all 52 estimated parameters of the EP DSGE model are shown in Figure 4. Table 2 reports the corresponding posterior summary statistics for all estimated parameters, split into several structural groups. In the remaining part of this subsection, the parameters in Table 2 are discussed in an order corresponding to their subjective economic importance.

[35] Diagnostic graphs for the individual parameters are available separately on request.

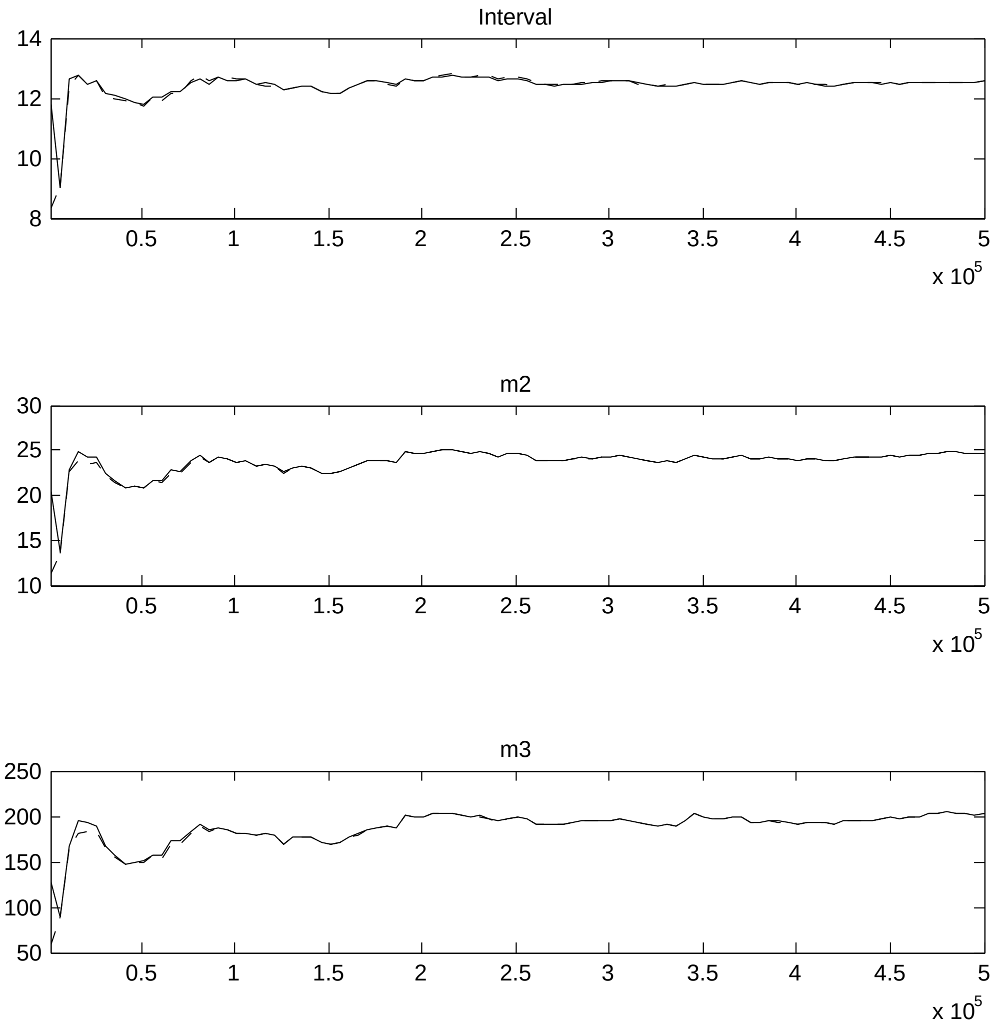

Figure 3: Solid and dashed lines reflect within and between chains recursive convergence diagnostics measures. The "interval" measure is computed using empirical 80% confidence intervals around the mean; the "m2" measure is based on the variance; and the "m3" measure is based on the third moment

The estimated posterior mean of ϕ_{fa}, that enters the country specific risk premium function $\Omega(FA_t, \epsilon_t^{\text{risk}})$ in (1), is equal to 0.0294. This value appears to be relatively low in comparison to some previously reported estimates in the literature. For example, in Adolfson et al. (2007a) the posterior mean of this parameter is 0.2520 in their benchmark euro area model.[36] The relatively low estimate of ϕ_{fa} in Table 2 may indicate that the net foreign asset position of Estonia cannot fully explain the observed interest rate spread between domestic and foreign interest rates. This is corroborated by the fact that the idiosyncratic component of the risk premium function, given by ϵ_t^{risk}, is very persistent, with the estimated posterior mean of ρ_{risk} equal to 0.8968, suggesting that it captures a high share of the risk premium variation in the data. This can partly be explained by data issues: Figure 2 clearly indicates the presence of two pronounced interest rate spikes in Estonian interest rates in the second half of the 1990s induced by the Asian and Russian financial crises. These events coincided with substantial structural shifts in the Estonian banking sector and a dramatic reduction in the interest rate spread in the following years. The specification of $\Omega(FA_t, \epsilon_t^{\text{risk}})$ function in (1) might be too parsimonious to describe these changes, and therefore may warrant a revision in future versions of the EP DSGE model.

The group of Calvo parameters reported in Table 2 carries information about the timing of price and wage setting decisions by firms and households (see Equations (21), (30), (31), and (32)). The plots of prior and posterior densities for θ_w, θ_p, θ_m and θ_e in Figure 4 reveal that the data has a lot to say about these parameters.

Posterior mean of θ_p, the parameter that governs degree of price stickiness faced by the domestic intermediate good producers, is estimated at 0.6376. This implies that domestic prices persist on average for about 2.75 quarters. This is somewhat lower than a recent survey evidence documented in Druant et al. (2009) who report an average price duration of 3.33 quarters for Estonia, even though their methodology does not make a distinction between domestic and consumer prices. Dabušinskas and Kulikov (2007) also find Calvo parameters being somewhat higher, in the range between 0.6830 and 0.8632, using three alternative specifications of the New Keynesian Phillips Curve estimated on a sample of Estonian domestic inflation data from 1994:4 to 2005:3. However, the conclusion of Dabušinskas and Kulikov (2007) that price setting in Estonia is more flexible than in the euro area still holds: most

[36]However, their results are subject to substantial variation. In another paper, which estimates a similar model on a long sample of Swedish data covering the period from 1980 to 2004, Adolfson et al. (2007b) report a posterior median of ϕ_{fa} in the range between 0.0310 to 0.0460 depending a particular model specification. It must be noted, that the risk premium function in that paper includes dependence on the expected exchange rate fluctuations, so that direct comparison to the EP DSGE model results is not possible.

empirical euro area DSGE models have substantially higher Calvo parameters associated with domestic price setting (see Smets and Wouters, 2003; Adolfson et al., 2007a).

Turning to the export-import sector, it is worth noting that the corresponding price stickiness parameters are considerably lower than θ_p. The implied price durations range from 1.34 quarters in the import sector (the posterior mean of θ_m is estimated at 0.2532) to just 1.13 quarters in the export sector (the posterior mean of θ_e is estimated at 0.1145). Adolfson et al. (2007a) also report lower price stickiness parameters in the export-import sector relative to the domestic one in their euro area model, though the implied durations according to their results range from 1.86 quarters for import prices to 2.77 quarters for export prices.

The posterior mean of the wage stickiness coefficient θ_w is estimated at 0.4965. According to this result, an average duration of a nominal wage contract in Estonia is about 2 quarters. This is twice lower than the survey evidence in Druant et al. (2009) for Estonia, where nominal wages are found to stay unchanged for a year on average. The corresponding result for the euro area also indicates a higher degree of wage stickiness, with an implied nominal wage durations of around 4 quarters (see Smets and Wouters 2003; and Adolfson et al., 2007a).

The next set of parameters in Table 2 is related to price and wage indexation. Coefficients τ_w, τ_p, τ_m and τ_e are linked to the weights of forward- and backward-looking inflation components in the real wage Equation (21) and corresponding New Keynesian Phillips Curves (30), (31), and (32). The prior-posterior density plots in Figure 4 suggest that the data is informative only about τ_w and τ_e indexation coefficients.

The posterior mean of the nominal wage indexation coefficient τ_w is estimated at 0.8617; it is noticeably higher than the corresponding euro area results documented by Smets and Wouters (2003) and Adolfson et al. (2007a). The estimated value of τ_w is difficult to put into perspective, because empirical evidence about wage indexation in Estonia is patchy. Druant et al. (2009) report that 34% of Estonian firms index wages to past inflation, but they do not quantify the degree of indexation adopted by the firms in their survey sample, making a direct comparison with the estimated τ_w coefficient problematical.

Another parameter of interest that can be calculated using empirical results in Table 2 links the real marginal cost term to the domestic inflation rate in the New Keynesian Phillips Curve Equation (30). This parameter, estimated at the posterior means of θ_p and τ_p, is equal to 0.1217. This is considerably higher than previously reported by Dabušinskas and Kulikov (2007), were a related coefficient was found to lie in the range from 0.0026 to 0.0113 depending on the model specification, but a formal statistical comparison of these results is infeasible due to differences in the estimation methodologies.

Other parameters of interest in Table 2 are following. The posterior mean of σ_c, the parameter governing inter-temporal elasticity of substitution of a representative household's consumption, is equal to 1.3302. It is in line with the value 1.3910 found by Smets and Wouters (2003) for the euro area, and implies that Estonian households respond to variations in the real interest rate in the same way as their European counterparts. The external consumption habit parameter h for Estonia is estimated at 0.8115, which exceeds both benchmark results reported for the euro area: 0.5920 in Smets and Wouters (2003) and 0.7080 in Adolfson et al. (2007a). It can be attributed to the "catching up with Joneses" effect that characterizes a country with high GDP growth rate such as Estonia.

The inverse elasticity of work effort with respect to the real wage is controlled by the parameter σ_l, with the posterior mean of 1.7988 reported in Table 2. The corresponding elasticity of labour supply with respect to the real wage is given by 0.5559, which is close to the result obtained in Staehr (2008), where he finds that a "1 per cent increase in after-tax hourly income would lead to a 0.6 percentage point increase in individuals being employed". On the other hand, Smets and Wouters (2003) find that the posterior mean of σ_l for the euro area is equal to 2.5030.

Two other elasticity parameters reported in Table 2 are η_c and η_*, which are respectively the elasticity of substitution between domestic and imported consumption goods in Estonia and between imported and domestic consumption goods in the euro area. Their estimated posterior means are very similar: 1.8678 for η_c and 1.7616 for η_*.[37] The pair of similar parameters for the euro area in Ratto et al. (2008) is reported to be 1.1724 and 2.5358 respectively, while Adolfson et al. (2007a) estimate the value of η_* at 1.4860.

Parameter φ that governs the elasticity of the investment adjustment cost function is estimated at 7.5716. The corresponding elasticity estimate is 0.1297, and according to the interpretation in Christiano et al. (2005), this implies that a 1% permanent change in the price of capital induces about a 13% change in investment. Similar parameter estimate obtained by Adolfson et al. (2007a) for the euro area is equal to 8.6700.

Finally, a pairwise comparison of estimated autoregressive parameters for Estonia and euro area in Table 2 reveals that the dynamics of structural shocks

[37]The typical estimates from micro-datasets for the elasticity of substitution between domestic and foreign goods range from 5 to 20 (refer to Obstfeld and Rogoff, 2000). However, macro-datasets usually yield much lower elasticity estimates, typically from 1.5 to 2 (e.g., see Collard and Dellas, 2002).

in both parts of the EP DSGE model is similar, except for the inter-temporal preference shocks $\widehat{\varepsilon}_t^{\beta}$ and $\widehat{\varepsilon}_t^{\beta,*}$. The posterior distributions of ρ_β for Estonia and ρ_β^* for the euro area shown in Figure 4 differ considerably, with the respective posterior means given by 0.2793 and 0.6285, the latter still being lower than the typical estimates obtained by Smets and Wouters (2003) and Adolfson et al. (2007a). An explanation of this finding might be related to the structure of shock transmission within the EP DSGE model: euro area structural shocks propagate to the Estonian economy, and therefore persistence of Estonian macroeconomic variables in part depends on the persistence of the euro area structural shocks. Hence, the inter-temporal preference shock ε_t^{β} can be less persistent without compromising the ability of the model to explain fluctuations of the main Estonian macroeconomic aggregates.

6.2. Model Fit

The Kalman filter representation of a typical DSGE model shown in (42) permits computation of the one-step ahead linear forecasts $\mathbb{E}(\tilde{\boldsymbol{y}}_t|\mathcal{F}_{t-1})$ based on the recursive information sets $\mathcal{F}_t := \{(\tilde{\boldsymbol{x}}_\tau, \tilde{\boldsymbol{y}}_\tau) : 1 \leq \tau \leq t\}$ (details can be found in Hamilton, 1994). This procedure corresponds to the usual idea of "goodness of fit" evaluation in dynamic time series models, where "fitted" values are compared to the actual data.[38]

This "goodness of fit" evaluation of the EP DSGE model is shown in Figure 5, where the one-step ahead linear Kalman filter forecasts evaluated at the posterior mode are plotted against all observables for Estonia and euro area throughout the full sample period. It follows that the empirical fit of the EP DSGE model is good for the euro area observables, and can informally be considered adequate for the Estonian macroeconomic variables. The model produces less satisfactory fit to the real wage, inflation and the the nominal interest rate series. Note that all these series are the most volatile in the sample, which can partly explain the lack of an adequate fit. Regarding the nominal interest rate series, there is a pronounced period where the model persistently "undershoots" the actual data, staring from 1999 and ending in 2006. This is caused by the cyclical fluctuations in the trade balance and the resulting cyclical increase in the net foreign asset position of Estonia over this time period, which roughly corresponds to the actual data.[39]

[38]There are several methods to validate an empirical DSGE model (refer to An and Schorfheide, 2007). Among them is the posterior odds comparison of an empirical DSGE with a Bayesian VAR model, and contrasting the autocorrelation-cross-correlation structure implied by an empirical DSGE model with that of the real-world data series. A more thorough model-validation exercise is left for the future revision of the EP DSGE model.

[39]Note that the net foreign asset position of Estonia has been steadily eroding over the

Table 2: The prior and posterior distributions of the EP DSGE model parameters

Parameter	Prior distribution			Posterior distribution			
	type	mean	std. deviation	mean	std. deviation	10%	90%
Standard deviation of shocks; Estonia							
Fiscal policy (σ_g)	*Inv. Gamma*	0.30	∞	0.2619	0.0859	0.1577	0.3885
Preference (σ_β)	*Inv. Gamma*	0.20	∞	0.0278	0.0029	0.0245	0.0316
Technology (σ_a)	*Inv. Gamma*	0.40	∞	0.0889	0.0145	0.0710	0.1094
Investment-specific technology (σ_x)	*Inv. Gamma*	0.10	∞	0.0736	0.0099	0.0610	0.0872
Risk premium (σ_{risk})	*Inv. Gamma*	0.08	∞	0.0113	0.0012	0.0099	0.0129
Wage mark-up (σ_w)	*Inv. Gamma*	0.25	∞	0.0344	0.0034	0.0306	0.0389
Domestic price mark-up (σ_p)	*Inv. Gamma*	0.15	∞	0.0238	0.0027	0.0203	0.0277
Import price mark-up (σ_m)	*Inv. Gamma*	0.40	∞	0.0881	0.0148	0.0690	0.1093
Export price mark-up (σ_e)	*Inv. Gamma*	0.40	∞	0.0990	0.0177	0.0757	0.1263
Standard deviation of shocks; Euro area							
Fiscal policy ($\sigma_{g,*}$)	*Inv. Gamma*	0.30	∞	0.2381	0.0219	0.2098	0.2683
Preference ($\sigma_{\beta,*}$)	*Inv. Gamma*	0.20	∞	0.0950	0.0142	0.0762	0.1158
Technology ($\sigma_{a,*}$)	*Inv. Gamma*	0.40	∞	0.3203	0.0299	0.2809	0.3615
Investment-specific technology ($\sigma_{x,*}$)	*Inv. Gamma*	0.10	∞	0.9168	0.1003	0.7860	1.0608
Wage mark-up ($\sigma_{w,*}$)	*Inv. Gamma*	0.25	∞	0.1073	0.0107	0.0935	0.1218
Price mark-up ($\sigma_{p,*}$)	*Inv. Gamma*	0.15	∞	0.0384	0.0047	0.0324	0.0449
Monetary policy ($\sigma_{r,*}$)	*Inv. Gamma*	0.10	∞	0.0693	0.0065	0.0610	0.0784

Notes: The EP DSGE model parameter name and symbol are shown in the first column. The following three columns describe the corresponding prior distribution: its type and the first two moments. In the case of an uniform prior, distribution's support is shown. The last four columns summarize the posterior distribution: its mean, standard deviation, and the 10th and 90th deciles. All posterior statistics are computed using the Metropolis-Hastings sampler.

Table 2: The prior and posterior distributions of the EP DSGE model parameters (cont.)

Parameter	Prior distribution			Posterior distribution			
	type	mean	std. deviation	mean	std. deviation	10%	90%
Auto-regressive coefficients; Estonia							
Fiscal policy (ρ_g)	*Beta*	0.85	0.10	0.8539	0.0679	0.7688	0.9322
Preference (ρ_β)	*Beta*	0.85	0.10	0.2793	0.0715	0.1933	0.3692
Technology (ρ_a)	*Beta*	0.85	0.10	0.6451	0.0872	0.5338	0.7550
Investment-specific technology (ρ_x)	*Beta*	0.85	0.10	0.5968	0.0900	0.4795	0.7094
Risk premium (ρ_{risk})	*Beta*	0.65	0.10	0.8968	0.0344	0.8484	0.9395
Wage mark-up (ρ_w)	*Beta*	0.85	0.10	0.3717	0.0982	0.2507	0.5022
Domestic price mark-up (ρ_p)	*Beta*	0.85	0.10	0.5895	0.1198	0.4466	0.7332
Import price mark-up (ρ_m)	*Beta*	0.85	0.10	0.8583	0.0699	0.7613	0.9439
Export price mark-up (ρ_e)	*Beta*	0.85	0.10	0.8298	0.0822	0.7318	0.9236
Auto-regressive coefficients; Euro area							
Fiscal policy (ρ^*_g)	*Beta*	0.85	0.10	0.8469	0.0655	0.7648	0.9228
Preference (ρ^*_β)	*Beta*	0.85	0.10	0.6285	0.0574	0.5512	0.7022
Technology (ρ^*_a)	*Beta*	0.85	0.10	0.7757	0.0698	0.6874	0.8615
Investment-specific technology (ρ^*_x)	*Beta*	0.85	0.10	0.4302	0.0722	0.3387	0.5222
Wage mark-up (ρ^*_w)	*Beta*	0.85	0.10	0.1785	0.0442	0.1243	0.2353
Price mark-up (ρ^*_p)	*Beta*	0.50	0.15	0.7134	0.0550	0.6411	0.7815
Monetary policy (ρ^*_m)	*Beta*	0.85	0.10	0.6998	0.0597	0.6183	0.7728

Notes: The EP DSGE model parameter name and symbol are shown in the first column. The following three columns describe the corresponding prior distribution: its type and the first two moments. In the case of an uniform prior, distribution's support is shown. The last four columns summarize the posterior distribution: its mean, standard deviation, and the 10th and 90th deciles. All posterior statistics are computed using the Metropolis-Hastings sampler.

Table 2: The prior and posterior distributions of the EP DSGE model parameters (cont.)

Parameter	Prior distribution			Posterior distribution			
	type	mean	std. deviation	mean	std. deviation	10%	90%
Calvo parameters; Estonia							
Wage (θ_w)	*Beta*	0.70	0.05	0.4965	0.0491	0.4341	0.5576
Domestic prices (θ_p)	*Beta*	0.75	0.05	0.6376	0.0441	0.5854	0.6894
Import prices (θ_m)	*Beta*	0.50	0.10	0.2532	0.0462	0.1970	0.3120
Export prices (θ_e)	*Beta*	0.50	0.10	0.1145	0.0242	0.0850	0.1457
Indexation parameters; Estonia							
Wage (τ_w)	*Beta*	0.75	0.15	0.8617	0.0763	0.7329	0.9636
Domestic prices (τ_p)	*Beta*	0.75	0.15	0.7387	0.1750	0.5476	0.9098
Import prices (τ_m)	*Beta*	0.50	0.15	0.5012	0.1689	0.3059	0.6990
Export prices (τ_e)	*Beta*	0.50	0.15	0.3655	0.1562	0.1936	0.5553
Elasticity parameters; Estonia							
Intertemporal substitution of consumption (σ_c)	*Normal*	1.00	0.375	1.3302	0.2997	0.9639	1.7140
Labour supply (σ_l)	*Normal*	2.00	0.25	1.7988	0.2686	1.4514	2.1418
Domestic-import substitution in Estonia (η_c)	*Inv. Gamma*	2.00	0.10	1.8678	0.1684	1.6698	2.0781
Import-domestic substitution in euro area (η_*)	*Inv. Gamma*	2.00	0.10	1.7616	0.1421	1.5857	1.9479
Capital utilization (ψ)	*Normal*	0.20	0.05	0.1694	0.0512	0.1035	0.2357
Investment adjustment cost (φ)	*Normal*	4.00	1.50	7.5716	1.1154	6.1969	8.9783

Notes: The EP DSGE model parameter name and symbol are shown in the first column. The following three columns describe the corresponding prior distribution: its type and the first two moments. In the case of an uniform prior, distribution's support is shown. The last four columns summarize the posterior distribution: its mean, standard deviation, and the 10th and 90th deciles. All posterior statistics are computed using the Metropolis-Hastings sampler.

Table 2: The prior and posterior distributions of the EP DSGE model parameters (cont.)

Parameter	Prior distribution			Posterior distribution			
	type	mean	std. deviation	mean	std. deviation	10%	90%
Other parameters; Estonia							
Share of fixed cost (ϕ)	*Normal*	0.45	0.08	0.1386	0.0796	0.0407	0.2374
Consumption habit (h)	*Beta*	0.70	0.05	0.8115	0.0341	0.7669	0.8535
Risk premium sensitivity (ϕ_{fa})	*Inv. Gamma*	0.50	∞	0.0294	0.0059	0.0219	0.0377
Consumer-domestic price ratio (γ^c)	*Inv. Gamma*	1.20	∞	1.1038	0.1021	1.0350	1.1777
Import-domestic price ratio (γ^m)	*Uniform*	1.00	1.30	1.1097	0.1449	1.0180	1.2242
Employment adjustment (χ)	*Beta*	0.50	0.15	0.8011	0.0205	0.7739	0.8275

Notes: The EP DSGE model parameter name and symbol are shown in the first column. The following three columns describe the corresponding prior distribution: its type and the first two moments. In the case of an uniform prior, distribution's support is shown. The last four columns summarize the posterior distribution: its mean, standard deviation, and the 10th and 90th deciles. All posterior statistics are computed using the Metropolis-Hastings sampler.

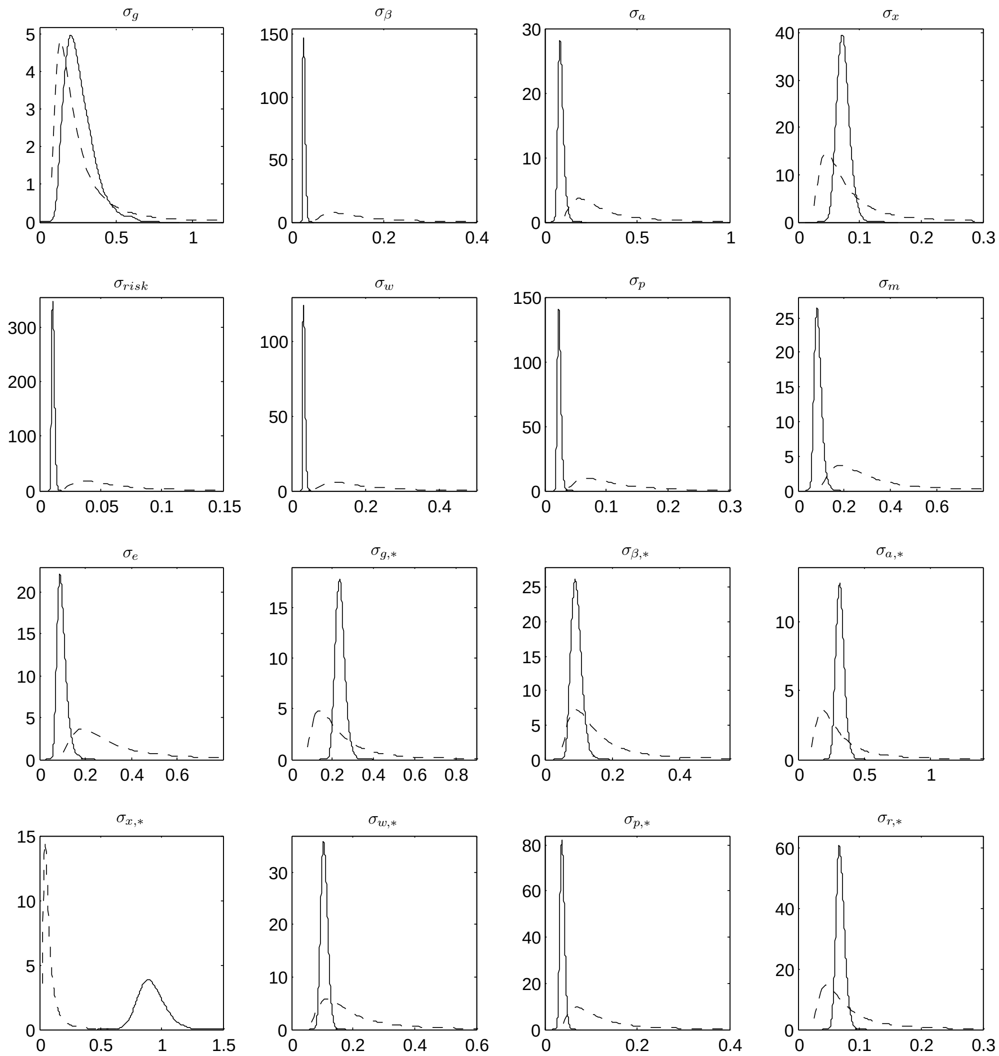

Figure 4: Prior distribution (dashed line) and estimated posterior distribution (solid line) of the EP DSGE model parameters

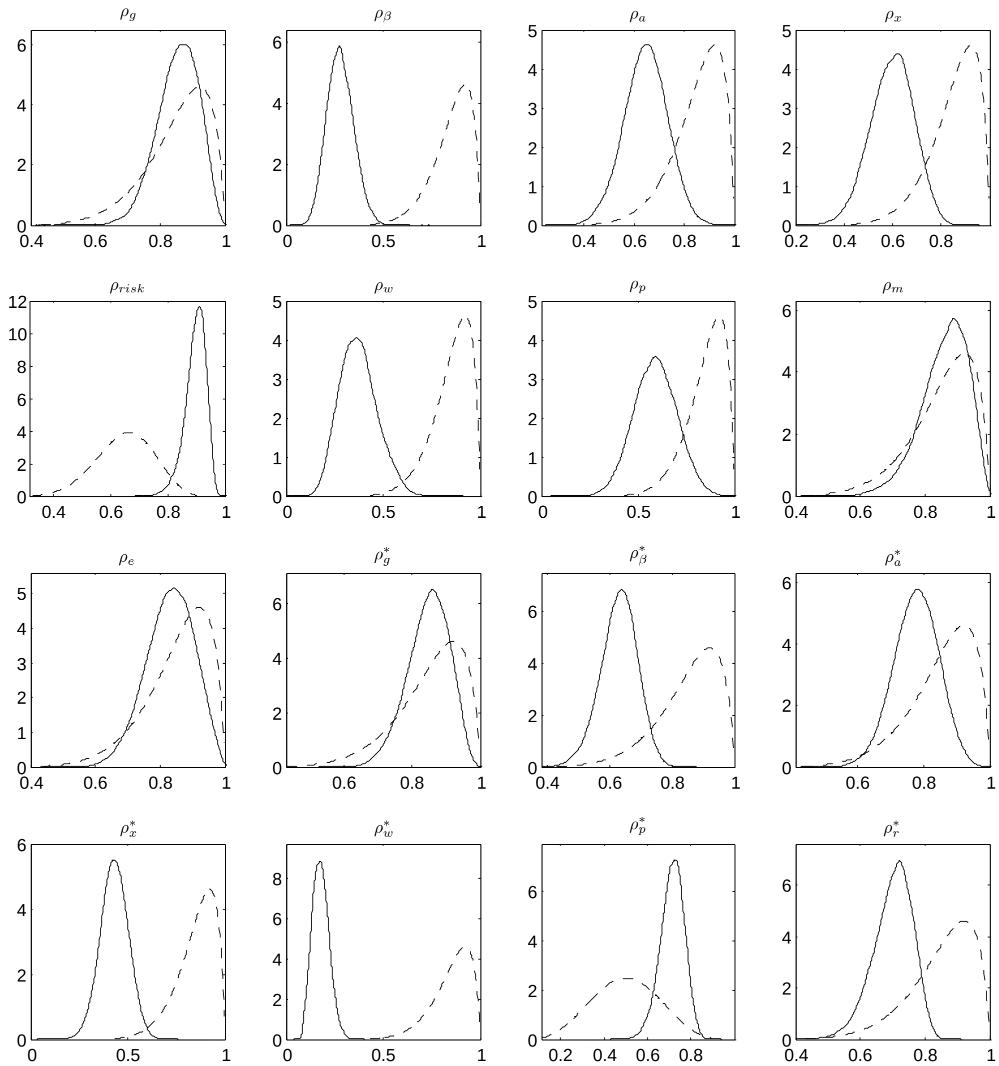

Figure 4: Prior distribution (dashed line) and estimated posterior distribution (solid line) of the EP DSGE model parameters (cont.)

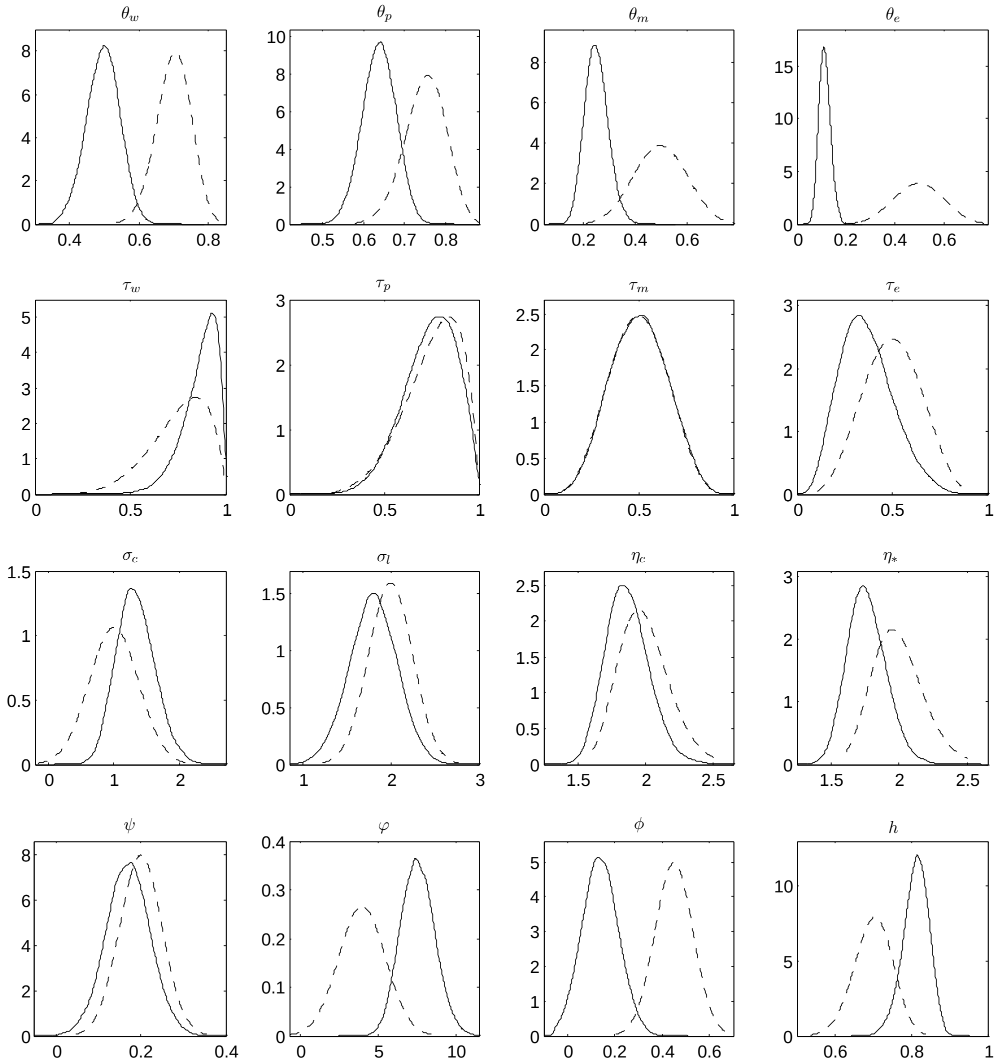

Figure 4: Prior distribution (dashed line) and estimated posterior distribution (solid line) of the EP DSGE model parameters (cont.)

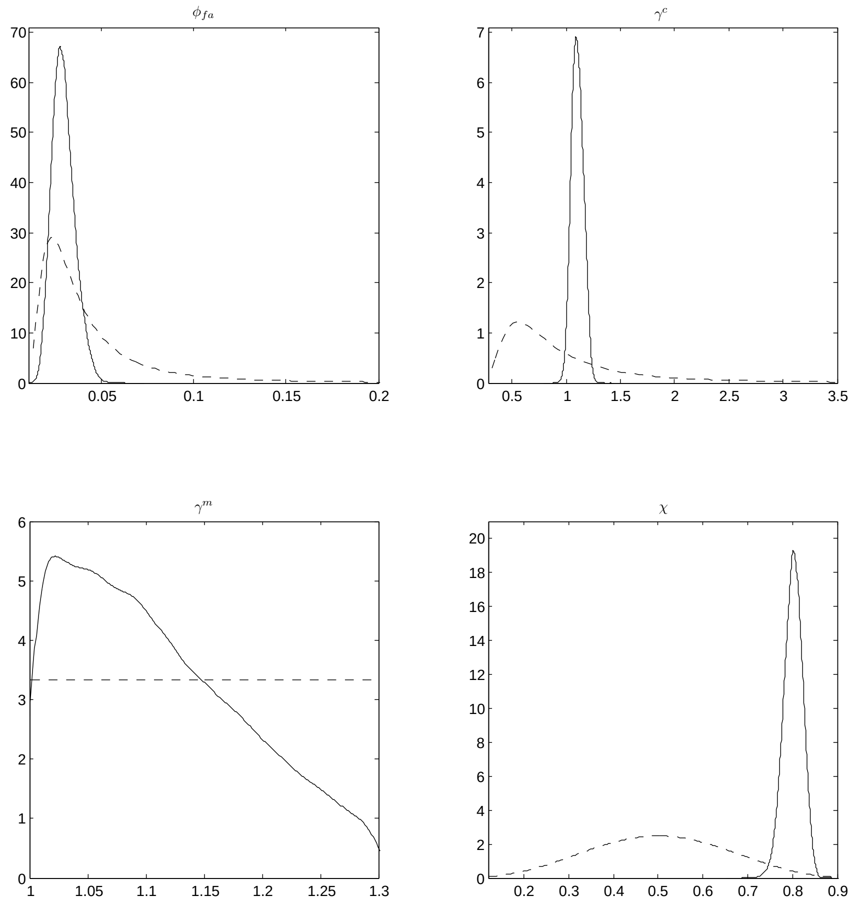

Figure 4: Prior distribution (dashed line) and estimated posterior distribution (solid line) of the EP DSGE model parameters (cont.)

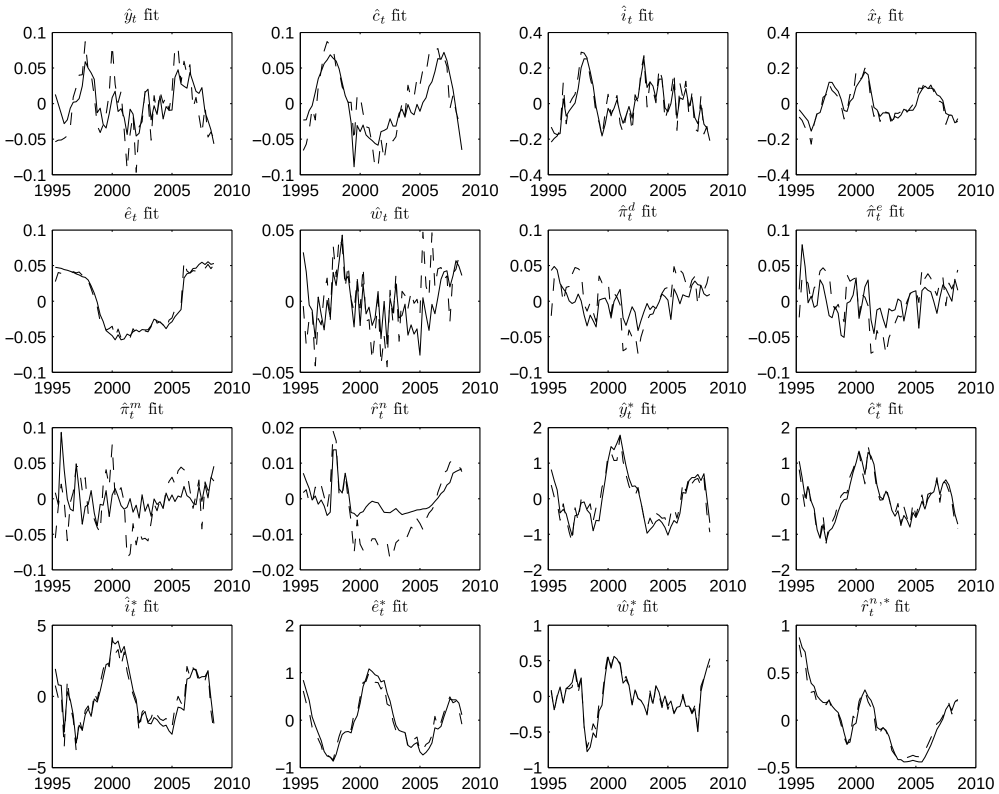

Figure 5: Actual data (solid line) and corresponding one-step ahead linear Kalman filter forecasts evaluated at the posterior mode (dotted line)

6.3. Model Response to Structural Shocks

Reaction of the endogenous model variables to structural shocks, such as changes in the risk premium, preferences or technology, can be examined using orthogonalized impulse response functions. This technique is commonly used in time series analysis and is based on the VAR representation of DSGE models shown in Subsection 5.1. For the estimated EP DSGE model, a collections of impulse response functions to some of the model's structural shocks are shown in Figures 6 to 14.[40] Note that these are responses to one standard deviation positive orthogonalized innovations to the vector of structural shocks z_t, computed using the VAR representation (41), where all model parameters are fixed at the corresponding posterior mode values and the variance-covarinace matrix Σ is transformed into a diagonal form (refer to Hamilton, 1994).

Figure 9 depicts a number of impulse responses to one standard deviation orthogonalized innovation to the risk premium shock $\widehat{\epsilon}_t^{\text{risk}}$. The set of endogenous variables on this figure, and all other figures showing the impulse response functions, contains central macroeconomic quantities related to both the domestic and euro area economy parts of the model. In order to understand the propagation of this shock through the model, one would ideally need to appeal to its effect on the real exchange rate, which in turns would have an impact on the export-import sector of the model. However, while the real exchange rate in not an explicit part of the EP DSGE model, the model contains two relative prices which are implicitly linked to the real exchange rate (see Equation (A.4) in Appendix 8.1).

A positive and unexpected risk premium shock generates an increase in both the nominal and real domestic interest rates, which in turn has a pronounced effects on the inter-temporal allocation decisions by the households. In particular, consumption and capital accumulation activity of the households decreases in favor of investing into bonds. This generates a demand-driven downturn in the domestic economy, with an associated decrease in the labour effort, real wage and the price of capital. On the production side, marginal cost drops and producer prices start to go down gradually through the standard mechanism of sticky prices. This leads to a depreciation of the real exchange rate and an associated rise in exports because of increased competitiveness *vis-à-vis* the euro area. At the same time, imports drop because of the lower

entire sample period, as the country borrowed funds needed to restructure its economy. The cyclical net foreign asset position refers to this data after removing the downward-sloping linear trend. However, de-trended net foreign asset position data series is not used for estimating the parameters of the EP DSGE model.

[40]Due to the space limitations only a subset of all impulse responses is reported in Figures 6 to 14. The full set of results is available from the authors on request.

domestic demand. The foreign asset position improves and the interest rate spread decreases to compensate for the initial risk premium shock. Note that domestic output rebounds sooner than both the consumption and investment because of the export-driven upturn. None of the euro area endogenous variables are affected by an idiosyncratic shock to $\widehat{\epsilon}_t^{\text{risk}}$, making it essentially one of the domestic shocks in the EP DSGE model.

Figure 7 depicts the model response to a one standard deviation unexpected orthogonalized innovation to the technology shock $\widehat{a}_t$. This shock hits the production side of the domestic economy: productivity of labour and capital rises uniformly for all intermediate good producers, leading to a decrease in marginal cost. A gradual process of producer price reduction ensues, and domestic inflation falls. The real exchange rate depreciates, leading to an increase in the exports to euro area. At the same time, imports fall initially, because of the substitution effect in favor of the domestic goods. The foreign asset position of Estonia improves, leading to a reduction of the risk premium and domestic nominal and real interest rates. As the result, households move from saving to consumption and capital accumulation, leading to an overall upturn in the economy. Note that the labour effort initially falls as a result of a positive technological innovation, a finding that is emphasized by Galì (1999) (and later corroborated by Smets and Wouters, 2005; Adolfson et al., 2007a; and others). A combination of sticky wages and prices drives the substitution from labour to capital at the point of initial impact of the technology shock, generating this result. The subsequent upturn in the economic activity drives up both the labour effort and the capital accumulation.

Impact of the investment-specific technology shock $\widehat{\epsilon}_t^x$ on the EP DSGE model is shown in Figure 8. The propagation mechanism of this shock is standard in DSGE literature; details can be found in Smets and Wouters (2003).

The model response to one standard deviation orthogonalized innovation to the domestic price mark-up shock $\widehat{\lambda}_t^p$ is shown in Figure 11. A positive shock reduces substitutability between the variety of differentiated domestically produced intermediate goods, and drives up the mark-ups charged by the intermediate good producers. As a result, domestic price inflation jumps up, leading to an appreciation of the real exchange rate and the initial drop in exports. At the same time, consumers shift from relatively more expensive domestic goods to the cheaper euro area imports. The net foreign asset position deteriorates, and the spread between domestic and euro area nominal interest rates increases. From this point on the response of main macroeconomic aggregates is similar to the previously described effect of a risk premium shock: reduced economic activity drives the marginal cost down, compensating for the initial jump in mark-ups, domestic prices start falling, the real exchange rate depreciates, and the exports rebound.

A positive unexpected wage mark-up shock, induced by one standard deviation orthogonalized innovation to $\widehat{\lambda}_t^w$, has a very similar effect on the domestic economy. The corresponding collection of impulse response functions is shown in Figure 10. The only difference with previously described effect of a price mark-up shock is that the marginal cost is driven up by an initial jump in real wages.[41]

The collection of impulse response functions describing an effect of one standard deviation orthogonalized innovation to the preference shock $\widehat{\epsilon}_t^\beta$ is depicted in Figure 6. A positive preference shock generates a boom-bust response of the domestic output in the EP DSGE model. The initial jump in output is due to an increased optimism of the households that shift from the capital accumulation to consumption. However, the drop in investment is so severe, that it quickly generates a bust in the overall level of economic activity. Employment, wages and domestic prices start to fall, and the real exchange rate depreciates. The latter leads to a subsequent pick-up in the euro area exports, generating a comeback in economic activity. At the same time, the initial jump in imports leads to a deterioration of the net foreign asset position and a gradual widening of the interest rate spread, which moderates the export-driven rebound.

The remaining part of this subsection covers the effect of three different euro area shocks on the Estonian economy. Recall that the EP DSGE model stipulates two types of links between domestic and euro area economy: the monetary policy channel via the UIP Equation (14) and the trading channel via the export-import sector described in Subsections 3.3 and 3.4. In addition, the structural shocks hitting domestic and euro area economy are assumed to be independent. One of the key assumptions of the EP DSGE model is that Estonian economy acts like a small open economy on the fringes of the big euro area economy: none of the domestic shocks in Figures 6 to 11 affect euro area macroeconomic variables.

Consider a monetary policy shock that hits the euro area economy; see Figure 14 for the corresponding collection of impulse response functions. A positive euro area monetary policy shock has well-known effects on its output and inflation; a detailed description of these effects can be found in Smets and

[41] The effect of a wage mark-up shock can be interpreted in different ways. If the shock is seen as a measure of how strong the trade unions are in the economy, wages rise in response to a positive wage mark-up shock because the unions have more bargaining power and can contract higher wages. According to the wage bargaining theory, this results in a lower overall employment in the economy. A different view on the wage mark-up shock is that it measures specific skills of different employees, and thereby substitutability of different types of labour in the economy. When a positive wage mark-up shock occurs, the labour demand described by (19) becomes less elastic, and the households that supply labour can earn a higher premium on their specific skills.

Wouters (2003). The model also generates an endogenous response of the euro area monetary policy authority to this shock; see Equation (38).

What are the effects of this shock on the Estonian economy? There are two effects which have a negative impact on the domestic output. Firstly, exports fall because of a decrease in the euro area output, at the same time imports from the euro area increase as they become relatively cheaper. Secondly, domestic nominal interest rate gradually increases because of the higher euro area interest rate and a deterioration in the net foreign asset position. Downturn in Estonian economy depresses consumption, wages and employment, as well as domestic prices. The latter helps to improve competitiveness *vis-à-vis* the euro area and has a moderating effect on the downturn.

Next, consider an euro area wage mark-up shock; the model responses are depicted in Figure 13. A positive wage mark-up shock effectively makes euro area less competitive *vis-à-vis* the Estonian economy. As a result, domestic economy experiences a boom driven by increased exports and the shift from relatively more expensive euro area imports to the domestically produced substitutes. Domestic investment, employment and wages increase, giving the households an extra income to spend on the consumption goods. Eventually, the net foreign asset position starts deteriorating due to the increased imports, and the interest rate spread widens, moderating the initial domestic economy boom.

The final euro area shock considered in this subsection is a one standard deviation orthogonalized positive innovation to $\widehat{a}_t^*$ shown in Figure 12. Its effect is essentially opposite to the previously described euro area wage mark-up shock: this time the euro area competitiveness improves *vis-à-vis* the Estonian economy. A foreign trade induced downturn in the domestic economic activity ensues, exaggerated by a rapid deterioration in the net foreign asset position and a resulting increase of the domestic nominal interest rate.

6.4. Variance Decomposition Analysis

The relative importance of various structural shocks included into the EP DSGE model can be measured by the share of total variation that a particular shock helps to explain for each endogenous variable of the model. The variance decomposition methodology is borrowed from the time series analysis (refer to Hamilton, 1994), and can be applied to DSGE models written in VAR form as shown by Equation (42).

Table 3 presents the variance decomposition results for the main endogenous variables of the EP DSGE model.[42] Full complement of 16 structural

[42]This results are produced by Dynare and are based on 20 lags approximation to the

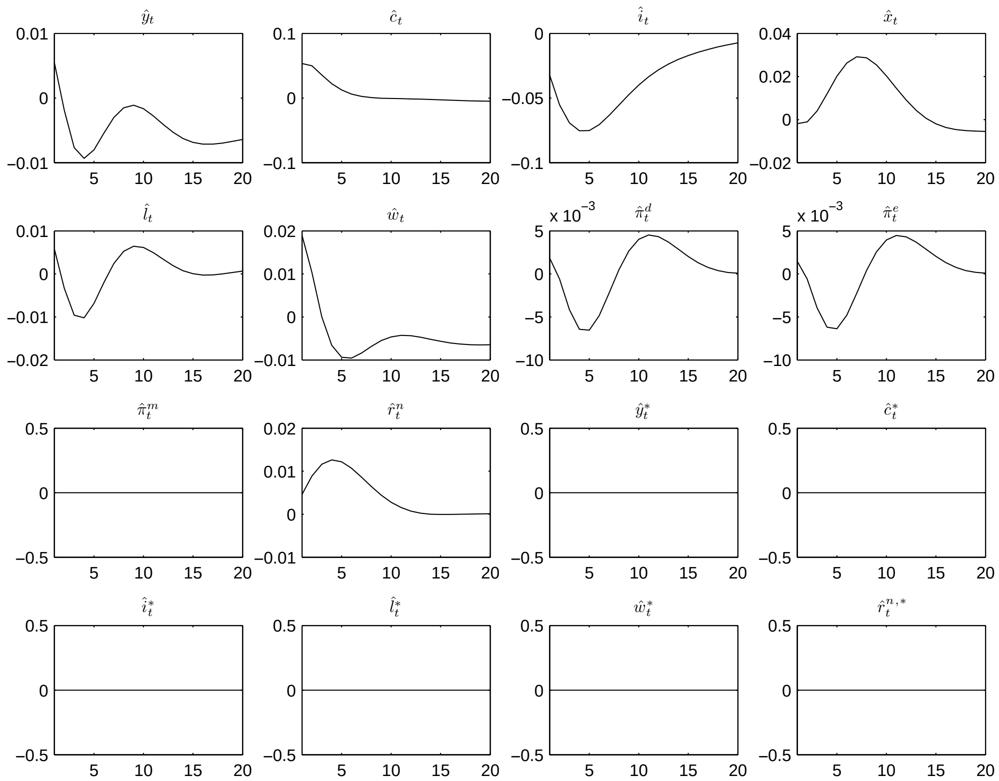

Figure 6: Impulse response functions (expressed in percentage deviations from the steady states) to one standard deviation orthogonalized innovation to $\widehat{\epsilon}_t^{\beta}$

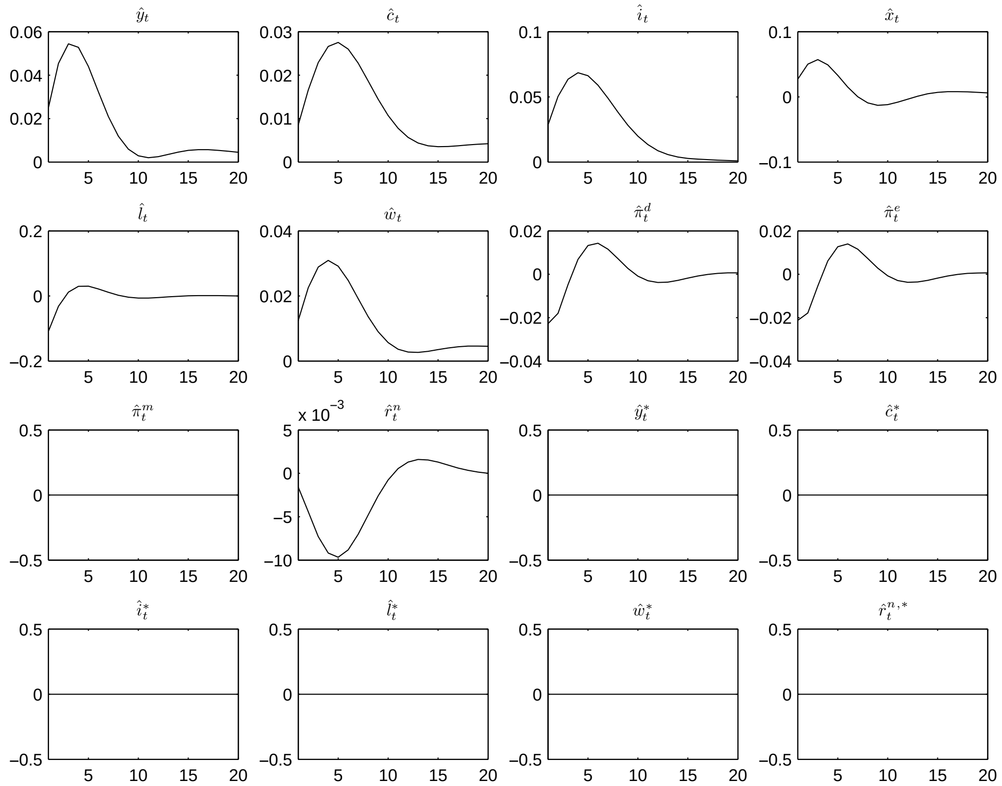

Figure 7: Impulse response functions (expressed in percentage deviations from the steady states) to one standard deviation orthogonalized innovation to $\widehat{a}_t$

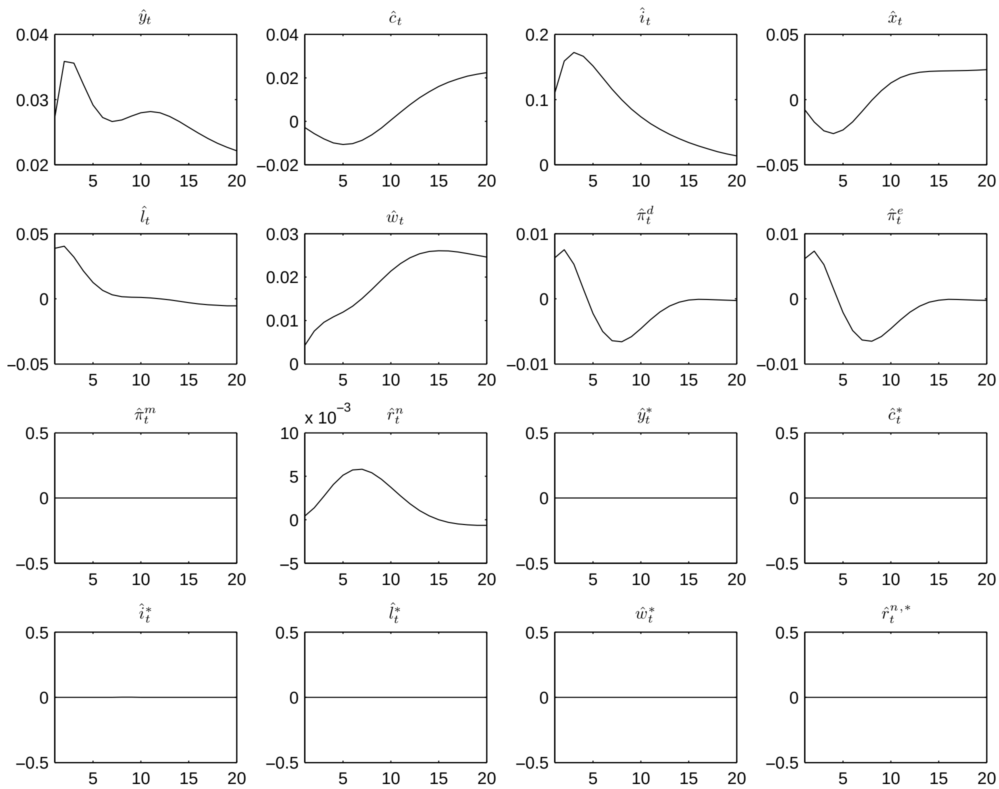

Figure 8: Impulse response functions (expressed in percentage deviations from the steady states) to one standard deviation orthogonalized innovation to $\widehat{\epsilon}_t^x$

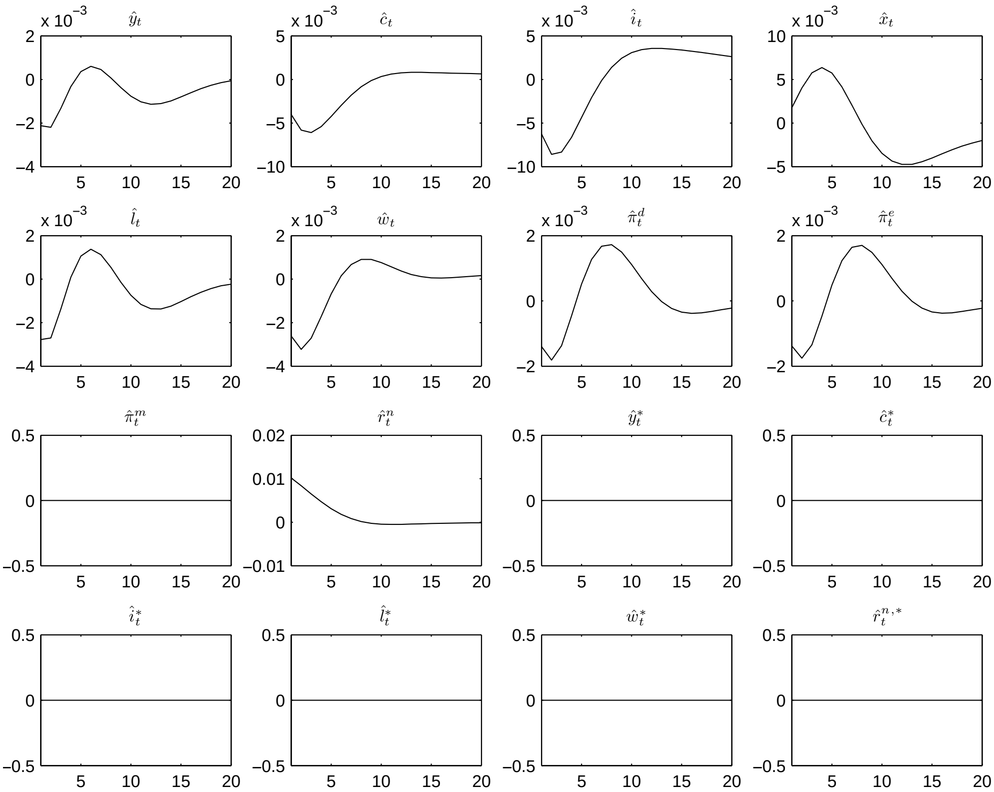

Figure 9: Impulse response functions (expressed in percentage deviations from the steady states) to one standard deviation orthogonalized innovation to $\widehat{\epsilon}_t^{\text{risk}}$

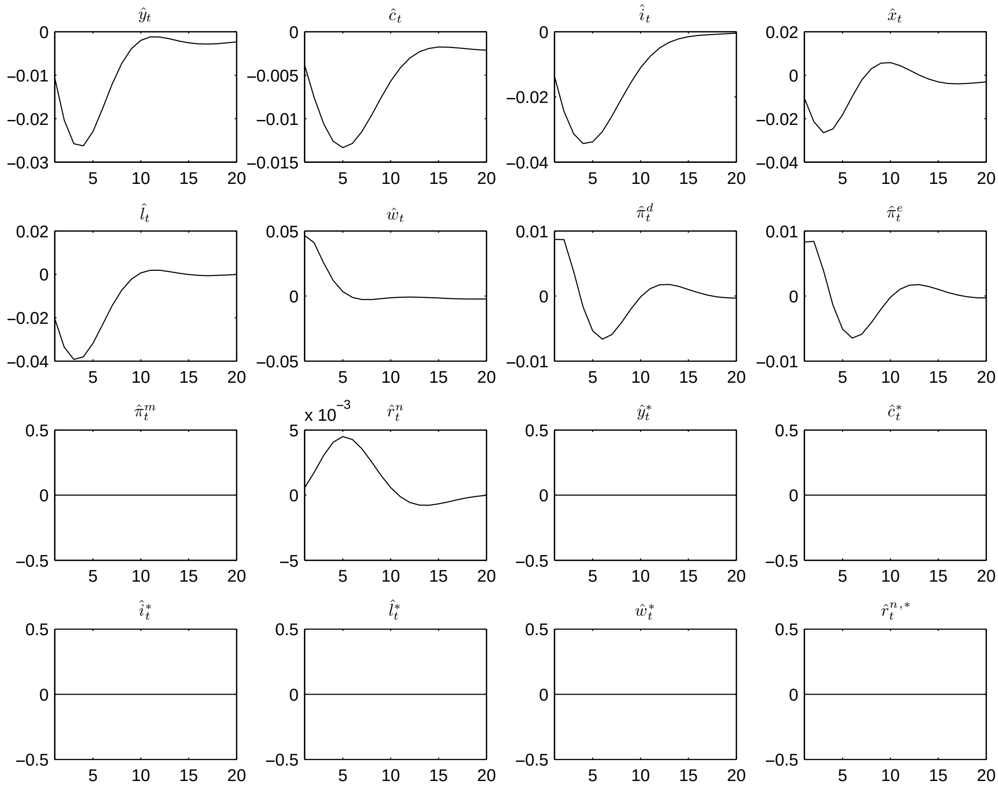

Figure 10: Impulse response functions (expressed in percentage deviations from the steady states) to one standard deviation orthogonalized innovation to $\widehat{\lambda}_t^w$

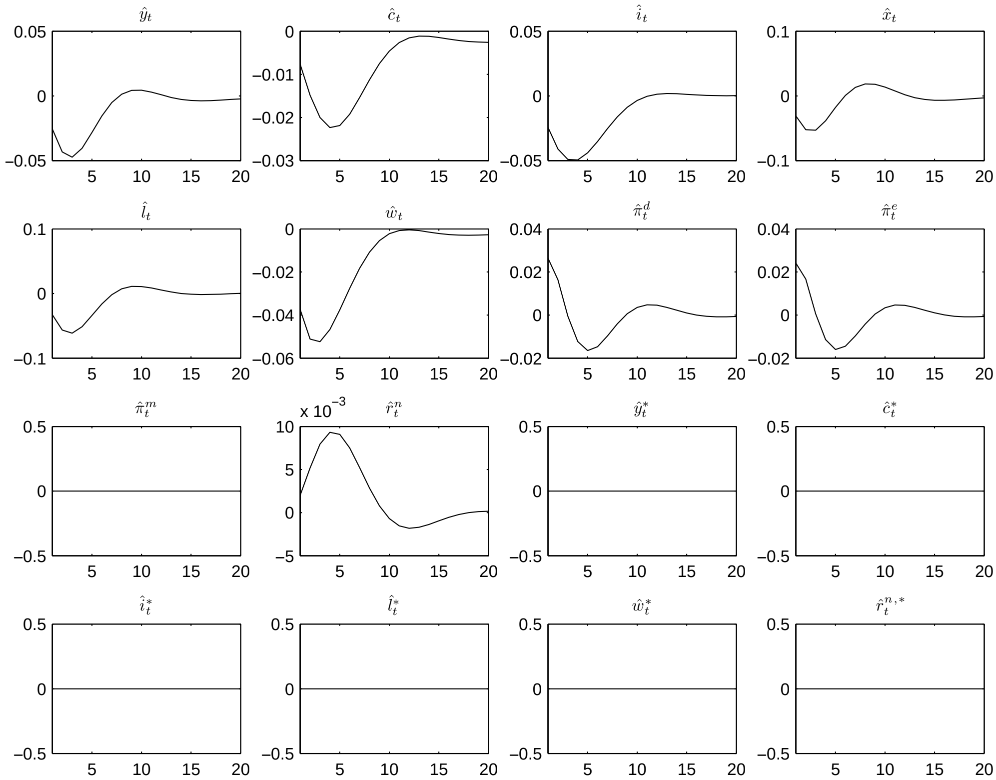

Figure 11: Impulse response functions (expressed in percentage deviations from the steady states) to one standard deviation orthogonalized innovation to $\widehat{\lambda}_t^p$

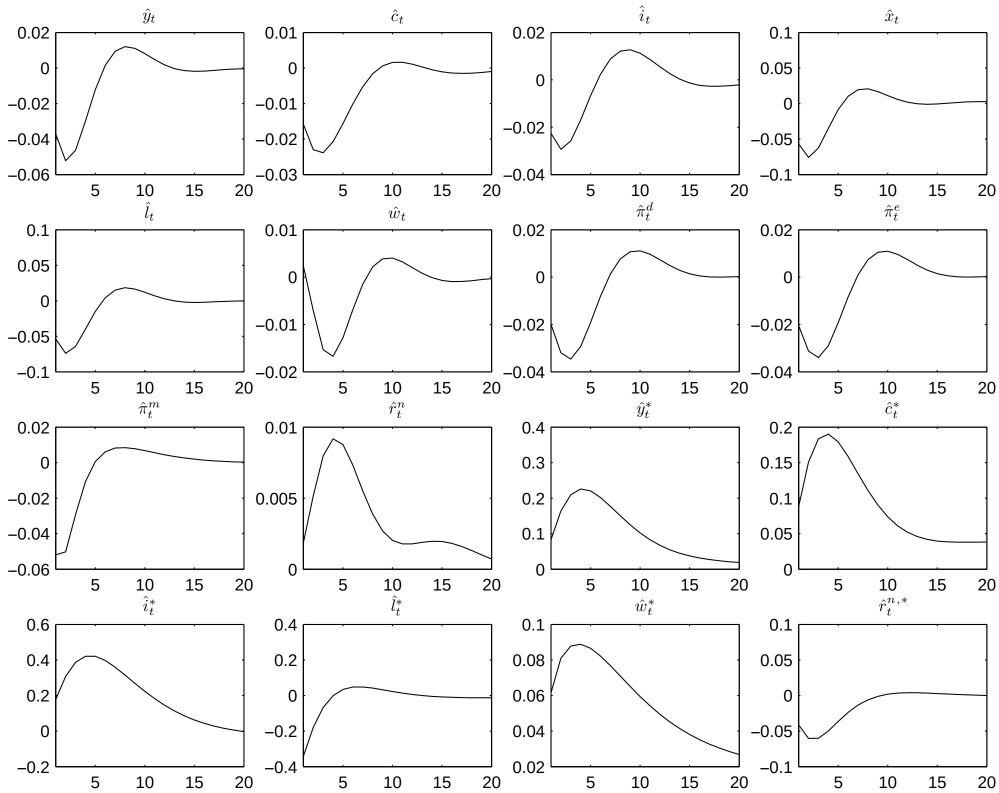

Figure 12: Impulse response functions (expressed in percentage deviations from the steady states) to one standard deviation orthogonalized innovation to $\widehat{a}_t^*$

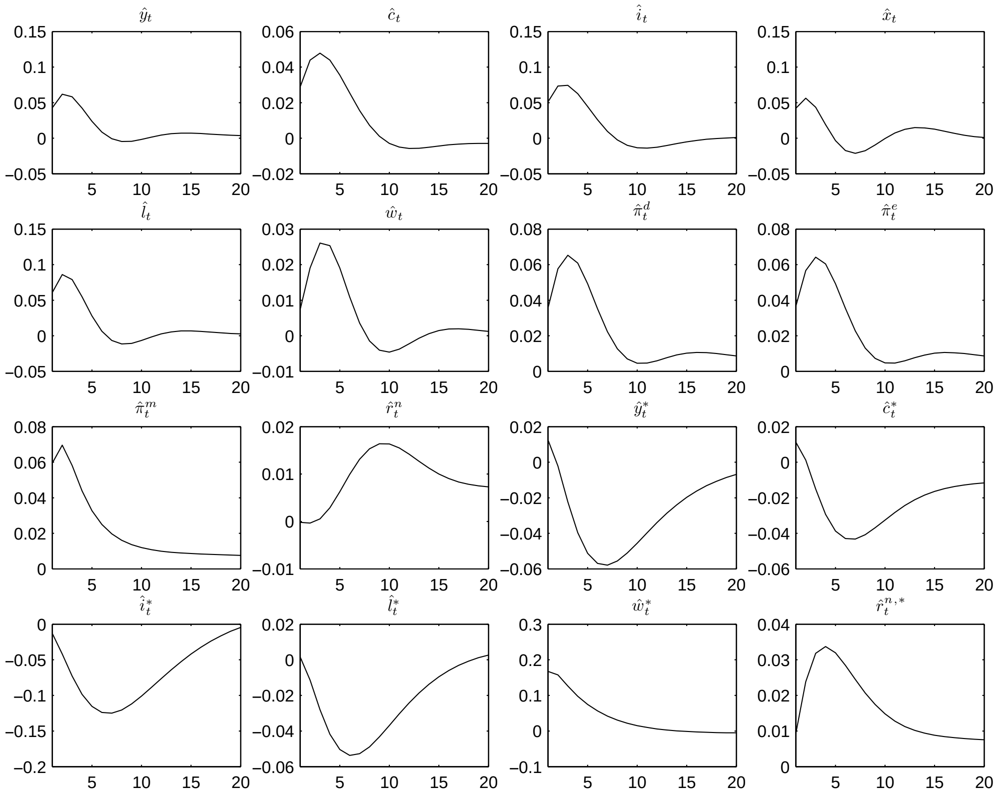

Figure 13: Impulse response functions (expressed in percentage deviations from the steady states) to one standard deviation orthogonalized innovation to $\widehat{\lambda}_t^{w,*}$

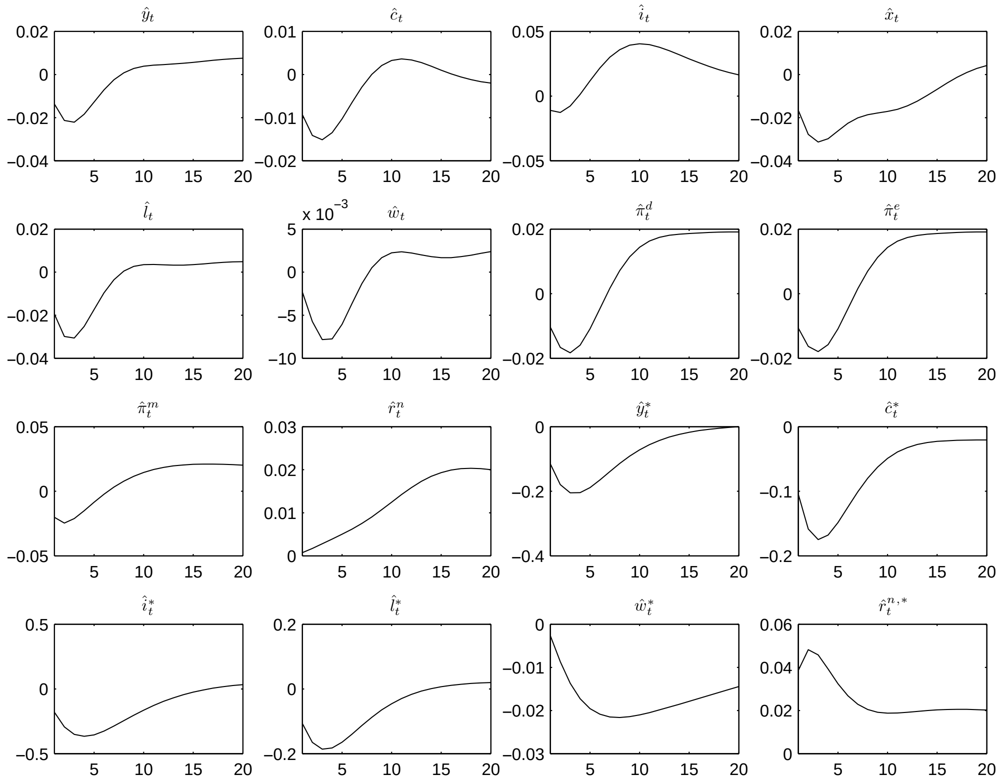

Figure 14: Impulse response functions (expressed in percentage deviations from the steady states) to one standard deviation orthogonalized innovation to $\widehat{\epsilon}_t^{r,*}$

innovations and the set of endogenous variables are divided into two groups: those related to the domestic economy and those linked to the euro area economy. The table can therefore be subdivided into four quadrants. The upper left quadrant shows a contribution of the domestic innovations to the Estonian economy. The results are mostly in line with the structural assumptions underlying most of the innovations, but some comments are still warranted. Two domestic innovations, the fiscal policy innovation u_t^g and the export price mark-up innovation u_t^e, appear to have a very limited contribution to the dynamics of model variables. Somewhat unexpectedly, the domestic price mark-up innovation u_t^p adds very little to the dynamics of domestic inflation $\widehat{\pi}_t^d$, but at the same time contributes significantly to other parts of the Estonian economy. The lower left quadrant of Table 3 is filled with zeros because Estonian innovation have no effect on euro area state variables.

Moving on to the contribution of euro area structural innovations, the lower right quadrant of Table 3 gives variance decomposition of the EP DSGE model's euro area part. This decomposition is similar to the results reported in Smets and Wouters (2003). The upper right quadrant of the table shows contribution of the euro area innovations to the Estonian economy. It can be observed that the euro area price and wage mark-up innovations, denoted $u_t^{p,*}$ and $u_t^{w,*}$ respectively, explain a significant share of the variation of most domestic endogenous variables. Given the small open economy nature of Estonia in the EP DSGE model, this result is not unexpected. However, these two innovations easily overwhelm the effect of most domestic innovations, which is especially noticeable in the case of domestic inflation $\widehat{\pi}_t^d$, real interest rate $\widehat{r}_t$, and price of capital $\widehat{q}_t$.

7. Conclusion

This paper works out the theoretical foundations and reports Bayesian estimations results for the first version of an open economy dynamic stochastic general equilibrium model for Estonia. The model is designed to match the key characteristics of the Estonian economy: the currency board regime, free capital mobility and dependence on the external economic environment via foreign trade. These are typical features of a small open economy in the vicinity of a much larger economic area. The EP DSGE model consists of two inter-dependent parts: the domestic economy part describing Estonia and the euro area part acting as a large closed economy with the monetary policy and trade linkages to the first part. The evolution of Estonian economy is described

unconditional variance-covariance matrix of the vector of state variables. Full variance decomposition results are available from the authors on demand.

Table 3: Variance decomposition of endogenous variables in estimated EP DSGE model (in percent)

	u_t^a	u_t^g	u_t^β	u_t^x	u_t^w	u_t^p	u_t^{risk}	u_t^m	u_t^e	$u_t^{p,*}$	$u_t^{w,*}$	$u_t^{r,*}$	$u_t^{\beta,*}$	$u_t^{a,*}$	$u_t^{x,*}$	$u_t^{g,*}$
$\widehat{y}_t$	15.22	1.24	0.49	4.52	3.59	12.10	0.03	1.03	0.11	23.55	19.27	3.00	0.36	15.15	0.08	0.25
$\widehat{c}_t$	7.73	0.48	25.30	4.14	1.80	5.46	0.46	0.70	0.01	13.52	27.96	3.29	0.03	6.58	2.44	0.10
$\widehat{i}_t$	8.70	0.67	9.49	49.01	2.21	4.96	0.19	0.33	0.00	4.64	12.40	3.05	0.06	2.21	2.06	0.03
$\widehat{r}_t$	3.45	0.08	2.44	0.96	0.77	3.57	0.84	0.83	0.02	26.47	39.03	4.93	0.15	14.64	1.60	0.23
$\widehat{r}_t^n$	9.81	0.26	14.65	3.24	2.13	9.32	7.01	2.68	0.06	9.05	15.79	6.73	0.80	6.06	12.31	0.11
$\widehat{\pi}_t^d$	6.10	0.14	0.78	1.06	1.22	7.20	0.07	0.78	0.04	24.18	35.51	6.39	0.25	14.04	2.02	0.23
$\widehat{\pi}_t^c$	1.76	0.04	0.22	0.30	0.35	2.07	0.02	2.32	0.01	27.94	36.21	8.19	0.45	17.50	2.35	0.28
$\widehat{k}_t$	2.73	0.30	3.51	85.82	0.71	1.31	0.05	0.11	0.00	0.75	2.56	1.17	0.02	0.33	0.62	0.01
$\widehat{l}_t$	17.84	1.17	0.49	3.83	4.31	10.70	0.03	1.12	0.11	23.31	18.87	2.65	0.35	14.95	0.04	0.25
$\widehat{q}_t$	5.89	0.16	5.88	5.05	1.30	5.60	0.60	0.72	0.02	20.28	36.98	4.39	0.07	10.40	2.50	0.16
$\widehat{w}_t$	11.87	0.10	4.05	1.65	21.55	36.85	0.15	1.51	0.02	7.13	9.53	0.98	0.04	3.98	0.53	0.07
$\widehat{r}_t^k$	13.88	0.76	1.04	4.21	2.01	22.30	0.07	0.78	0.09	20.07	19.59	2.36	0.23	12.26	0.14	0.20
$\widehat{fa}_t$	10.22	0.27	15.27	3.38	2.22	9.71	1.69	2.79	0.06	9.79	16.42	7.20	0.94	7.09	12.80	0.15
$\widehat{\pi}_t^m$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	6.14	0.00	30.93	30.65	8.15	0.66	20.88	2.28	0.32
$\widehat{\pi}_t^x$	5.83	0.13	0.75	1.04	1.17	6.74	0.07	0.77	0.93	24.09	35.56	6.41	0.25	13.97	2.05	0.22
$\widehat{x}_t$	11.58	0.35	3.23	4.03	2.54	11.46	0.27	3.42	0.68	26.31	11.40	2.82	0.81	18.62	2.14	0.35
$\widehat{m}_t$	10.30	0.23	23.33	2.11	2.15	11.16	1.15	5.57	0.22	10.81	16.17	2.01	0.53	6.94	7.20	0.12
$\widehat{c}_t^*$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.47	0.82	13.28	57.04	14.98	10.29	3.12
$\widehat{i}_t^*$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.55	0.64	6.31	11.55	7.97	71.70	1.28
$\widehat{r}_t^{k,*}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	7.07	8.43	17.20	5.19	21.78	33.85	6.48
$\widehat{r}_t^{n,*}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.00	7.00	12.77	35.36	29.11	10.05	2.71
$\widehat{\pi}_t^*$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	34.19	30.98	8.14	0.72	23.27	2.34	0.36
$\widehat{w}_t^*$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	21.54	55.75	0.61	0.98	16.26	4.78	0.08
$\widehat{l}_t^*$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.01	1.46	18.72	8.41	34.16	26.46	9.78
$\widehat{y}_t^*$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.35	1.60	20.29	11.18	22.65	31.63	11.30

by 25 state variables and 9 structural shocks, while the euro area part consists of 13 state variables and 7 structural shocks.

The empirical part of this paper reports Bayesian estimation results, impulse response functions and variance decomposition of the main endogenous variables of the model. Out of 59 structural parameters in the EP DSGE model, 52 are estimated using a data sample consisting of 16 macroeconomic series for Estonia and euro area. The main empirical findings are largely in line with previous studies for Estonia, whenever a direct comparison can be made. It is worth mentioning that the net foreign asset position of Estonia is found to be an economically significant factor in explaining the country risk premium in the UIP equation, but the empirical results suggest that other explanatory factors may also be warranted.

The empirical relevance of structural shocks in the model is assessed by the variance decomposition of the main endogenous variables. It is found that three most important domestic shocks in explaining the variability of Estonian macroeconomic series are the consumption preference shock and two technology shocks. Euro area shocks also play a prominent role in driving the dynamics of Estonian macroeconomic aggregates. Among the most significant euro area shocks that affect Estonia are the price and wage mark-up shocks.

The first version of the EP DSGE model described in this paper is focused on the business cycle frequency fluctuations of the main Estonian macroeconomic aggregates, leaving their long-run trends aside. Future developments of the model are likely to incorporate the long-run dynamics as well, considering that Estonia is still experiencing the effects of real and nominal convergence as it catches up with the developed euro area economies. Other potential future extensions of the model include incorporation of the financial sector with the associated frictions, integration of the housing sector together with collateral-constrained households, and expansion of the government sector.

8. Appendix

8.1. Relative Prices, Marginal Costs and the Real Exchange Rate

A number of relative prices and marginal cost terms used in the EP DSGE model are defined in this appendix, together with their log-linearized versions. In addition, the real exchange rate is defined as well.

Let the ratio of consumer prices to domestic prices in the Estonian economy be denoted by γ_t^c. The definition and the corresponding log-linearized version of γ_t^c are given by:

$$\gamma_t^c := \frac{P_t^c}{P_t^d}\,, \qquad \widehat{\gamma}_t^c = \widehat{p}_t^c - \widehat{p}_t^d\,.$$

By adding and subtracting $\widehat{p}_{t-1}^c$ and $\widehat{p}_{t-1}^d$ on the left-hand side of the last expression, one obtains:

$$\widehat{\gamma}_t^c = \widehat{\pi}_t^c - \widehat{\pi}_t^d + \widehat{\gamma}_{t-1}^c\,.$$

Let γ_t^e denote the ratio of export prices of Estonian goods to the euro area prices. Then:

$$\gamma_t^e := \frac{P_t^e}{P_t^*}\,, \qquad \widehat{\gamma}_t^e = \widehat{p}_t^e - \widehat{p}_t^*\,, \qquad \widehat{\gamma}_t^e = \widehat{\pi}_t^e - \widehat{\pi}_t^* + \widehat{\gamma}_{t-1}^e\,. \tag{A.1}$$

Let γ_t^* denote the relative producer price in the Estonian economy to the corresponding euro area price. Then $\gamma_t^* = MC_t^e \gamma_t^e$ by the definition of MC_t^e in Footnote 25 and γ_t^e in Equation (A.1):

$$\gamma_t^* := \frac{P_t^d}{\overline{e}\,P_t^*} = MC_t^e \gamma_t^e\,, \qquad \widehat{\gamma}_t^* = \widehat{p}_t^d - \widehat{p}_t^* = \widehat{mc}_t^e + \widehat{\gamma}_t^e\,. \tag{A.2}$$

The log-linearized version of MC_t^e defined in Footnote 25 is given by $\widehat{mc}_t^e = \widehat{p}_t^d - \widehat{p}_t^e$, from where:

$$\widehat{mc}_t^e = \widehat{\pi}_t^d - \widehat{\pi}_t^e + \widehat{mc}_{t-1}^e\,.$$

Let γ_t^m denote the ratio of import prices to domestic prices in the Estonian economy:

$$\gamma_t^m := \frac{P_t^m}{P_t^d}\,, \qquad \widehat{\gamma}_t^m = \widehat{p}_t^m - \widehat{p}_t^d\,, \qquad \widehat{\gamma}_t^m = \widehat{\pi}_t^m - \widehat{\pi}_t^d + \widehat{\gamma}_{t-1}^m\,. \tag{A.3}$$

Using the definition of MC_t^m in Footnote 24 together with Equation (A.2):

$$\widehat{mc}_t^m = \widehat{p}_t^* - \widehat{p}_t^m = \widehat{p}_t^d - \widehat{mc}_t^e - \widehat{\gamma}_t^e - \widehat{p}_t^m\,,$$

from where by definition of $\widehat{\gamma}_t^m$ in Equation (A.3) follows:

$$\widehat{mc}_t^m = -\widehat{mc}_t^e - \widehat{\gamma}_t^e - \widehat{\gamma}_t^m\,.$$

Finally, definition of the real exchange rate e_t^* in terms of the relative consumer prices is given by:

$$e_t^* := \frac{\overline{e}\,P_t^*}{P_t^c}\,.$$

Multiplying it by $\frac{P_t^d}{P_t^d}$ and using the definition of γ_t^* and γ_t^c leads to the following expression:

$$e_t^* = \frac{\overline{e}\,P_t^*}{P_t^c}\frac{P_t^d}{P_t^d} = \frac{1}{\gamma_t^* \gamma_t^c}\,. \tag{A.4}$$

An increase (decrease) in either γ_t^* or γ_t^c or both implies a decrease (increase) of the real exchange rate, which means an appreciation (depreciation) given the previous definition.

8.2. The Log-Linearized Model for Estonia

The aggregate resource constraint:

$$\widehat{y}_t = (1-\alpha_c)(\gamma^c)^{\eta_c}\frac{C}{Y}(\widehat{c}_t + \eta_c \widehat{\gamma}_t^c) + \frac{I}{Y}\widehat{i}_t + \frac{G}{Y}\widehat{g}_t + \frac{X}{Y}\widehat{x}_t\,;$$

Export dynamics:

$$\widehat{x}_t = (\gamma^e)^{-\eta_*}\frac{Y^*}{X}(\widehat{y}_t^* - \eta_* \widehat{\gamma}_t^e)\,;$$

Import dynamics:

$$\widehat{m}_t = \frac{C}{M}\left[\widehat{c}_t - \eta_c(1-\alpha_c)(\gamma^c)^{-(1-\eta_c)}\widehat{\gamma}_t^m\right];$$

The net foreign assets law of motion:[43]

$$\begin{aligned}\frac{1}{Y}\widehat{fa}_t =& \frac{Y^*}{Y}\left[\widehat{y}_t^* - \eta_*\widehat{\gamma}_t^e - \widehat{mc}_t^e\right] + \frac{M}{Y}\widehat{\gamma}_t^* \\ & - \frac{M}{Y}\left[\widehat{c}_t - \eta_c(1-\alpha_c)(\gamma^c)^{-(1-\eta_c)}\widehat{\gamma}_t^m\right] + \frac{1}{Y}R^n\,\widehat{fa}_{t-1}\,;\end{aligned}$$

The Fisher equation:

$$\widehat{r}_t = \widehat{r}_t^n - \mathbb{E}_t\,\widehat{\pi}_{t+1}^c\,;$$

The consumption Euler equation:

$$\widehat{c}_t = \frac{h}{1+h}\widehat{c}_{t-1} + \frac{1}{1+h}\mathbb{E}_t\,\widehat{c}_{t+1} - \frac{1-h}{\sigma_c(1+h)}\widehat{r}_t + \widehat{\varepsilon}_t^\beta\,;$$

[43]The steady-state assumptions under which this equation is derived are the following: $FA = 0$, $\log\Omega(FA, 1) = 0$, $\gamma^e = \gamma^* = MC^e = 1$, $R^n = R^{n,*}$. Steady-state values of other parameters are shown in Appendix 8.4..

Real wage dynamics:

$$\begin{aligned}\widehat{w}_t =& \frac{\beta}{1+\beta}\,\mathbb{E}_t\,\widehat{w}_{t+1} + \frac{1}{1+\beta}\widehat{w}_{t-1} + \frac{\beta}{1+\beta}\,\mathbb{E}_t\,\widehat{\pi}^c_{t+1} - \frac{1+\beta\tau_w}{1+\beta}\widehat{\pi}^c_t + \frac{\tau_w}{1+\beta}\widehat{\pi}^c_{t-1} \\ & - \frac{1}{1+\beta}\frac{(1-\beta\theta_w)(1-\theta_w)}{\left(1+\frac{1+\lambda_w}{\lambda_w}\sigma_l\right)\theta_w}\left[\widehat{w}_t - \sigma_l\widehat{l}_t - \frac{\sigma_c}{1-h}(\widehat{c}_t - h\widehat{c}_{t-1})\right] + \widehat{\lambda}^w_t\,;\end{aligned}$$

The households' investment decision equation, where $\varphi := S''(1)$ is elasticity of the investment adjustment cost function S:

$$\widehat{i}_t = \frac{1}{1+\beta}\widehat{i}_{t-1} + \frac{\beta}{1+\beta}\,\mathbb{E}_t\,\widehat{i}_{t+1} + \frac{1}{1+\beta}\frac{1}{\varphi}\,\widehat{q}_t + \widehat{\varepsilon}^x_t\,;$$

Price of capital dynamics:

$$\widehat{q}_t = -\widehat{r}_t + \frac{1-\delta}{1-\delta+R}\,\mathbb{E}_t\,\widehat{q}_{t+1} + \frac{R}{1-\delta+R}\,\mathbb{E}_t\,\widehat{r}^k_{t+1}\,;$$

The production function, where $\psi := \frac{\Psi'(1)}{\Psi''(1)}$ is the inverse elasticity of the capital utilization cost function Ψ, and ϕ is the share of fixed cost in production:

$$\frac{1}{1+\phi}\,\widehat{y}_t = \widehat{a}_t + \alpha\,\widehat{k}_{t-1} + \alpha\psi\,\widehat{r}^k_t + (1-\alpha)\,\widehat{l}_t\,;$$

labour demand:[44]

$$\widehat{l}_t = -\widehat{w}_t + (1+\psi)\,\widehat{r}^k_t + \widehat{k}_{t-1}\,;$$

Employment adjustment equation:

$$\widehat{e}_t = \frac{\beta}{1+\beta}\,\mathbb{E}\,\widehat{e}_{t+1} + \frac{1}{1+\beta}\,\widehat{e}_{t-1} + \frac{1}{1+\beta}\frac{(1-\beta\chi)(1-\chi)}{\chi}(\widehat{l}_t - \widehat{e}_t)\,;$$

Marginal cost dynamics:

$$\widehat{mc}_t = \alpha\,\widehat{r}^k_t + (1-\alpha)\,\widehat{w}_t - \widehat{a}_t\,;$$

[44] Log-linearized form of Equation (6) is given by:

$$R^k(1+\widehat{r}^k_t) = \Psi'(1) + \Psi''(1)\,\widehat{z}_t\,.$$

Note that in the steady state Equation (6) satisfies $R^k = \Psi'(z) = \Psi'(1)$. Hence:

$$\widehat{z}_t = \frac{\Psi'(1)}{\Psi''(1)}\,\widehat{r}^k_t = \psi\,\widehat{r}^k_t\,.$$

Log-linearized form of Equation (26) is given by:

$$\widehat{l}_t = -\widehat{w}_t + \widehat{r}^k_t + \widehat{z}_t + \widehat{k}_{t-1}\,.$$

Combining the two previous equations leads to the labour demand function shown in the text.

The capital accumulation equation:

$$\widehat{k}_t = \delta\big(\widehat{i}_t + \varphi\,\widehat{\varepsilon}_t^x\big) + (1-\delta)\,\widehat{k}_{t-1}\,;$$

Domestic inflation dynamics is given by the New Keynesian Phillips Curve:

$$\widehat{\pi}_t^d = \frac{\beta}{1+\beta\tau_p}\,\mathbb{E}_t\,\widehat{\pi}_{t+1}^d + \frac{\tau_p}{1+\beta\tau_p}\,\widehat{\pi}_{t-1}^d + \frac{1}{1+\beta\tau_p}\frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p}\,\widehat{mc}_t + \widehat{\lambda}_t^p\,;$$

The inflation dynamics of imported consumption goods:

$$\widehat{\pi}_t^m = \frac{\beta}{1+\tau_m\beta}\,\mathbb{E}_t\,\widehat{\pi}_{t+1}^m + \frac{\tau_m}{1+\tau_m\beta}\,\widehat{\pi}_{t-1}^m + \frac{1}{1+\tau_m\beta}\frac{(1-\theta_m)(1-\beta\theta_m)}{\theta_m}\widehat{mc}_t^m + \widehat{\lambda}_t^m\,;$$

The inflation dynamics of exported consumption goods:

$$\widehat{\pi}_t^e = \frac{\beta}{1+\tau_e\beta}\,\mathbb{E}_t\,\widehat{\pi}_{t+1}^e + \frac{\tau_e}{1+\tau_e\beta}\,\widehat{\pi}_{t-1}^e + \frac{1}{1+\tau_e\beta}\frac{(1-\theta_e)(1-\beta\theta_e)}{\theta_e}\widehat{mc}_t^e + \widehat{\lambda}_t^e\,;$$

The marginal cost of imported consumption goods:

$$\widehat{mc}_t^m = -\widehat{mc}_t^e - \widehat{\gamma}_t^e - \widehat{\gamma}_t^m\,;$$

The marginal cost of exported consumption goods:

$$\widehat{mc}_t^e = \widehat{\pi}_t^c - \widehat{\pi}_t^e + \widehat{mc}_{t-1}^e\,;$$

The ratio of consumer prices to domestic prices:[45]

$$\widehat{\gamma}_t^c = \widehat{\pi}_t^c - \widehat{\pi}_t^d + \widehat{\gamma}_{t-1}^c\,;$$

The ratio of import prices to domestic prices:[45]

$$\widehat{\gamma}_t^m = \widehat{\pi}_t^m - \widehat{\pi}_t^d + \widehat{\gamma}_{t-1}^m\,;$$

The ratio of export prices to the euro area consumer prices:[45]

$$\widehat{\gamma}_t^e = \widehat{\pi}_t^e - \widehat{\pi}_t^* + \widehat{\gamma}_{t-1}^e\,;$$

The ratio of domestic to euro area prices:[45]

$$\widehat{\gamma}_t^* = \widehat{mc}_t^e + \widehat{\gamma}_t^e\,;$$

Consumer price inflation:

$$\widehat{\pi}_t^c = (1-\alpha_c)(\gamma^c)^{1-\eta_c}\,\widehat{\pi}_t^d + \alpha_c(\gamma^m)^{1-\eta_c}\,\widehat{\pi}_t^m\,;$$

[45] Refer to Appendix 8.1 for details.

The monetary policy equation, also known as the modified UIP condition:

$$\widehat{r}_t^n = \widehat{r}_t^{n,*} - \phi_{\text{fa}}\, \widehat{fa}_t + \widehat{\varepsilon}_t^{\text{risk}}\,;$$

Exogenous fiscal policy:[46]

$$\widehat{g}_t = \rho_g \widehat{g}_{t-1} + u_t^g\,, \qquad u_t^g \sim \text{WN}(0, \sigma_g^2)\,;$$

Structural shocks:

Preference:	$\widehat{\varepsilon}_t^\beta = \rho_\beta\, \widehat{\varepsilon}_{t-1}^\beta + u_t^\beta,\ u_t^\beta \sim \text{WN}(0, \sigma_\beta^2);$
Technology:	$\widehat{a}_t = \rho_a\, \widehat{a}_{t-1} + u_t^a,\ u_t^a \sim \text{WN}(0, \sigma_a^2);$
Investment-specific technology:	$\widehat{\varepsilon}_t^x = \rho_x\, \widehat{\varepsilon}_{t-1}^x + u_t^x,\ u_t^x \sim \text{WN}(0, \sigma_x^2);$
Risk premium:	$\widehat{\varepsilon}_t^{\text{risk}} = \rho_{\text{risk}}\, \widehat{\varepsilon}_{t-1}^{\text{risk}} + u_t^{\text{risk}},\ u_t^{\text{risk}} \sim \text{WN}(0, \sigma_{\text{risk}}^2);$
Wage mark-up:	$\widehat{\lambda}_t^w - \lambda^w = \rho_w(\widehat{\lambda}_{t-1}^w - \lambda^w) + u_t^w,\ \ u_t^w \sim \text{WN}(0, \sigma_w^2);$
Domestic price mark-up:	$\widehat{\lambda}_t^p - \lambda^p = \rho_p(\widehat{\lambda}_{t-1}^p - \lambda^p) + u_t^p,\ \ u_t^p \sim \text{WN}(0, \sigma_p^2);$
Import price mark-up:	$\widehat{\lambda}_t^m = \rho_m\, \widehat{\lambda}_{t-1}^m + u_t^m,\ \ u_t^m \sim \text{WN}(0, \sigma_m^2);$
Export price mark-up:	$\widehat{\lambda}_t^e = \rho_e\, \widehat{\lambda}_{t-1}^e + u_t^e,\ \ u_t^e \sim \text{WN}(0, \sigma_e^2).$

8.3. The Log-Linearized Model for the Euro Area[47]

The aggregate resource constraint:

$$\widehat{y}_t^* = \frac{C^*}{Y^*}\widehat{c}_t^* + \frac{I^*}{Y^*}\widehat{i}_t^* + \frac{G^*}{Y^*}\widehat{g}_t^* + \frac{K^*}{Y^*}R^{k,*}\psi^*\, \widehat{r}_t^{k,*}\,;$$

The Fisher equation:

$$\widehat{r}_t^* = \widehat{r}_t^{n,*} - \mathbb{E}_t\, \widehat{\pi}_{t+1}^*\,;$$

---

[46]Fiscal policy in the model is given by equaton (34):

$$\log G_t = \rho_g \log G_{t-1} + u_t^g\,.$$

By substituting G_t with its definition in terms of the percentage deviation from its steady-state G, i.e. $G_t = G(1 + \widehat{g}_t)$, the equation becomes:

$$\log G(1 + \widehat{g}_t) = \rho_g \log G(1 + \widehat{g}_{t-1}) + u_t^g\,,$$

and its log-linear approximation is given by:

$$\widehat{g}_t = \rho_g \widehat{g}_{t-1} + u_t^g\,.$$

Other exogenous shock processes in the model are approximated around their corresponding steady states in the similar manner.

[47]Refer to Smets and Wouters (2003) for a detailed coverage of the euro area model. The inflation objective, price of capital and labour supply shocks are missing in this version of their model. The relative risk aversion parameter is set to unity.

The consumption Euler equation:

$$\widehat{c}_t^* = \frac{h^*}{1+h^*}\widehat{c}_{t-1}^* + \frac{1}{1+h^*}\,\mathbb{E}_t\,\widehat{c}_{t+1}^* - \frac{1-h^*}{1+h^*}\widehat{r}_t^* + \widehat{\varepsilon}_t^{\beta,*}\,;$$

Real wage dynamics:

$$\widehat{w}_t^* = \frac{\beta^*}{1+\beta^*}\,\mathbb{E}_t\,\widehat{w}_{t+1}^* + \frac{1}{1+\beta^*}\widehat{w}_{t-1}^* + \frac{\beta^*}{1+\beta^*}\,\mathbb{E}_t\,\widehat{\pi}_{t+1}^* - \frac{1+\beta^*\tau_w^*}{1+\beta^*}\widehat{\pi}_t^* + \frac{\tau_w^*}{1+\beta^*}\widehat{\pi}_{t-1}^*$$
$$- \frac{1}{1+\beta^*}\frac{(1-\beta^*\theta_w^*)(1-\theta_w^*)}{\left(1+\frac{1+\lambda_w^*}{\lambda_w^*}\sigma_l^*\right)\theta_w^*}\left[\widehat{w}_t^* - \sigma_l^*\widehat{l}_t^* - \frac{1}{1-h^*}(\widehat{c}_t^* - h^*\widehat{c}_{t-1}^*)\right] + \widehat{\lambda}_t^{w,*}\,;$$

The household investment decision equation, where $\varphi^* := S''(1)$ is elasticity of the investment adjustment cost function S:

$$\widehat{i}_t^* = \frac{1}{1+\beta^*}\widehat{i}_{t-1}^* + \frac{\beta^*}{1+\beta^*}\mathbb{E}_t\,\widehat{i}_{t+1}^* + \frac{1}{1+\beta^*}\frac{1}{\varphi^*}\,\widehat{q}_t^* + \widehat{\varepsilon}_t^{x,*}\,;$$

Price of capital dynamics:

$$\widehat{q}_t^* = -\,\widehat{r}_t^* + \frac{1-\delta^*}{1-\delta^*+R^*}\mathbb{E}_t\,\widehat{q}_{t+1}^* + \frac{R^*}{1-\delta^*+R^*}\mathbb{E}_t\,\widehat{r}_{t+1}^{k,*}\,;$$

The production function, where $\psi^* := \frac{\Psi'(1)}{\Psi''(1)}$ is the inverse of the elasticity of the capital utilization cost function and ϕ^* is the share of fixed cost in production:

$$\frac{1}{1+\phi^*}\,\widehat{y}_t^* = \widehat{a}_t^* + \alpha^*\,\widehat{k}_{t-1}^* + \alpha^*\psi^*\,\widehat{r}_t^{k,*} + (1-\alpha^*)\,\widehat{l}_t^*\,;$$

labour demand:[48]

$$\widehat{l}_t^* = -\widehat{w}_t^* + (1+\psi^*)\widehat{r}_t^{k,*} + \widehat{k}_{t-1}^*\,;$$

Employment adjustment equation:

$$\widehat{e}_t^* = \frac{\beta^*}{1+\beta^*}\,\mathbb{E}\,\widehat{e}_{t+1}^* + \frac{1}{1+\beta^*}\,\widehat{e}_{t-1}^* + \frac{1}{1+\beta^*}\frac{(1-\beta^*\chi^*)(1-\chi^*)}{\chi^*}(\widehat{l}_t^* - \widehat{e}_t^*)\,;$$

Marginal cost dynamics:

$$\widehat{mc}_t^* = \alpha^*\,\widehat{r}_t^{k,*} + (1-\alpha^*)\,\widehat{w}_t^* - \widehat{a}_t^*\,;$$

The capital accumulation equation:

$$\widehat{k}_t^* = \delta^*\left(\widehat{i}_t^* + \varphi^*\,\widehat{\varepsilon}_t^{x,*}\right) + (1-\delta^*)\,\widehat{k}_{t-1}^*\,;$$

[48] See Footnote 44 for derivations details.

Euro area inflation dynamics is given by the New Keynesian Phillips Curve:

$$\widehat{\pi}_t^* = \frac{\beta^*}{1+\beta^*\tau_p^*}\,\mathbb{E}_t\,\widehat{\pi}_{t+1}^* + \frac{\tau_p^*}{1+\beta^*\tau_p^*}\widehat{\pi}_{t-1}^* + \frac{1}{1+\beta^*\tau_p^*}\,\frac{(1-\beta^*\theta_p^*)(1-\theta_p^*)}{\theta_p^*}\widehat{mc}_t^* + \widehat{\lambda}_t^{p,*}\,;$$

The monetary policy rule:[49]

$$\begin{aligned}\widehat{r}_t^{n,*} = &\phi_m\,\widehat{r}_{t-1}^{n,*} + (1-\phi_m)\big[r_\pi \widehat{\pi}_{t-1}^* + r_y(\widehat{y}_{t-1}^* - \widehat{y}_{t-1}^{p,*})\big]\\ &\qquad + r_{\Delta\pi}(\widehat{\pi}_t^* - \widehat{\pi}_{t-1}^*) + r_{\Delta y}\big[\widehat{y}_t^* - \widehat{y}_t^{p,*} - (\widehat{y}_{t-1}^* - \widehat{y}_{t-1}^{p,*})\big] + \widehat{\varepsilon}_t^{r,*}\,;\end{aligned}$$

Exogenous fiscal policy:

$$\widehat{g}_t^* = \rho_g^*\,\widehat{g}_{t-1}^* + u_t^{g,*}\,, \qquad u_t^{g,*} \sim \text{WN}(0,\sigma_{g,*}^2)\,;$$

Structural shocks:

Preference:	$\widehat{\varepsilon}_t^{\beta,*} = \rho_\beta^*\,\widehat{\varepsilon}_{t-1}^{\beta,*} + u_t^{\beta,*},\ \ u_t^{\beta,*} \sim \text{WN}(0,\sigma_{\beta,*}^2)$;
Technology:	$\widehat{a}_t^* = \rho_a^*\,\widehat{a}_{t-1}^* + u_t^{a,*},\ \ u_t^{a,*} \sim \text{WN}(0,\sigma_{a,*}^2)$;
Investment-specific technology:	$\widehat{\varepsilon}_t^{x,*} = \rho_x^*\,\widehat{\varepsilon}_{t-1}^{x,*} + u_t^{x,*},\ \ u_t^{x,*} \sim \text{WN}(0,\sigma_{x,*}^2)$;
Wage mark-up:	$\widehat{\lambda}_t^{w,*} - \lambda_w^* = \rho_w^*(\widehat{\lambda}_{t-1}^{w,*} - \lambda_w^*) + u_t^{w,*},\ \ u_t^{w,*} \sim \text{WN}(0,\sigma_{w,*}^2)$;
Price mark-up:	$\widehat{\lambda}_t^{p,*} = \rho_p^*\,\widehat{\lambda}_{t-1}^{p,*} + u_t^{p,*},\ \ u_t^{p,*} \sim \text{WN}(0,\sigma_{p,*}^2)$;
Monetary policy:	$\widehat{\varepsilon}_t^{r,*} = \rho_r^*\,\widehat{\varepsilon}_{t-1}^{r,*} + u_t^{r,*},\ \ u_t^{r,*} \sim \text{WN}(0,\sigma_{r,*}^2)$.

8.4. The Steady State

In this section the steady state values of the main EP DSGE model endogenous variables are derived analytically.

Let the interest rate parity hold in the steady state:

$$R^n = R^{n,*}\,.$$

Then it follows from Equation (14) that:

$$\Omega(FA,1) = 1\,,$$

which, given $\log\Omega(FA,1) = -\phi_{\text{fa}}\,FA$ and the previous equation, implies that in the steady state:

$$FA = B^* = 0\,. \tag{A.5}$$

The steady state value of the return on capital R^k can be obtained from Equation (13) under the conditions of no investment adjustment cost, steady-state capital utilization rate $z = 1$, and $\Psi(1) = 0$:

$$R^k = R^n - 1 + \delta\,.$$

[49] The potential output level is denoted by $\widehat{y}_t^{p,*}$; refer to Section 4 for details.

Keeping in mind that $R^n = \frac{1}{\beta}$ because of the zero inflation steady state, one can write the steady-state return on capital R^k as:

$$R^k = \beta^{-1} - 1 + \delta\,. \tag{A.6}$$

Steady-state aggregate real profit is given by:

$$\Pi = (1 + \lambda^p)Y - R^k K - \frac{W}{P^d}L - \Phi\,,$$

where λ^p is is the steady-state mark-up parameter defined in Subsection 3.2.1, and Φ is the aggregate real fixed cost term (see Subsection 3.2.2). In equilibrium, the entire income is divided between capital and labour shares $Y = R^k K + \frac{W}{P^d}L$, but due to the fixed cost the steady-state real profit is zero

$$0 = \Pi = (1 + \lambda^p)Y - Y - \Phi = \lambda^p Y - \Phi\,,$$

from where:

$$\Phi = \lambda^p Y\,. \tag{A.7}$$

Using the fact that in the steady state all firms have the same production plans and use the same technology (23), the previous equation can be written as:

$$\Phi = \lambda^p \left[\left(\frac{K}{L} \right)^\alpha L - \Phi \right]\,.$$

Solving for Φ gives the following expression for the steady-state fixed cost parameter:

$$\Phi = \frac{\lambda^p}{1 + \lambda^p} \left(\frac{K}{L} \right)^\alpha L\,. \tag{A.8}$$

Finally, combining Equation (A.7) with (A.8) gives the steady state output Y:

$$Y = \frac{1}{1 + \lambda^p} \left(\frac{K}{L} \right)^\alpha L\,. \tag{A.9}$$

From Equation (35), the steady-state capital utilization rate $z = 1$, and $\Psi(1) = 0$, the aggregate resource constraint in the steady state is given by:

$$Y = C^d + C^m + I + G + X - M\,,$$

which by Equation (36) reduces to:

$$Y = C^d + I + G + X\,. \tag{A.10}$$

Now, given the steady-state value of the foreign assets position in (A.5), Equation (37) implies that real imports and exports are equal in the steady state:

$$X = M = C^m = \alpha_c \left(\frac{P^m}{P^c}\right)^{-\eta_c} C\,,$$

where the last equality comes from Equation (17). Multiplying by $\left(\frac{P^d}{P^d}\right)^{-\eta_c}$ and using definitions of γ^c and γ^m in Appendix 8.1:

$$X = \alpha_c \left(\frac{\gamma^c}{\gamma^m}\right)^{\eta_c} C\,. \tag{A.11}$$

Next, from Equation (16) the steady-state level of domestic consumption is:

$$C^d = (1-\alpha_c)\left(\frac{P^d}{P^c}\right)^{-\eta_c} C\,,$$

from where by definition of γ^c in Appendix 8.1:

$$C^d = (1-\alpha_c)(\gamma^c)^{\eta_c}\, C\,. \tag{A.12}$$

Substituting Equations (A.11) and (A.12) into (A.10) and keeping in mind that the steady-state investment is given by $I = \delta K$ by Equation (2), the aggregate resource constraint in the steady state can be written as:

$$Y = (1-\alpha_c)(\gamma^c)^{\eta_c}\, C + \delta K + \frac{G}{Y}\, Y + \alpha_c \left(\frac{\gamma^c}{\gamma^m}\right)^{\eta_c} C\,.$$

Letting $g := \frac{G}{Y}$, and re-arranging and solving for C gives:

$$[(1-\alpha_c)(\gamma^m)^{\eta_c} + \alpha_c]\left(\frac{\gamma^c}{\gamma^m}\right)^{\eta_c} C = (1-g)Y - \delta K\,.$$

Now, letting $\xi := [(1-\alpha_c)(\gamma^m)^{\eta_c} + \alpha_c]\left(\frac{\gamma^c}{\gamma^m}\right)^{\eta_c}$, the previous expression becomes:[50]

$$C = \frac{1}{\xi}\big[(1-g)\,Y - \delta\, K\big]\,.$$

Finally, using Equation (A.9), the steady state consumption level in terms of $\frac{K}{L}$ and L is:

$$C = \frac{1}{\xi}\left[\frac{1-g}{1+\lambda^p}\left(\frac{K}{L}\right)^{\alpha} - \delta\,\frac{K}{L}\right] L\,. \tag{A.13}$$

[50] Note, that in the closed-economy case $\xi = 1$ because the share of imports in consumption is zero ($\alpha_c = 0$) and the CPI is equal to the domestic price index ($\gamma^c = 1$).

An expression for $\frac{K}{L}$ is needed. Because in the steady state there is no price stickiness and other inefficiencies, the steady-state value of the real marginal cost is given by the inverse of the mark-up (refer to Footnote 22):

$$MC = \frac{1}{1+\lambda^p}\,.$$

The steady-state version of Equation (27) combined with the previous expression gives the following:

$$\frac{1}{1+\lambda^p} = \left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}(R^k)^{\alpha}\left(\frac{W}{P^d}\right)^{1-\alpha}\,.$$

Solving for $\frac{W}{P^d}$:

$$\frac{W}{P^d} = \left[(1+\lambda^p)\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}(R^k)^{\alpha}\right]^{-\frac{1}{1-\alpha}}\,. \tag{A.14}$$

The steady-state version of Equation (26) is given by:

$$\frac{K}{L} = \frac{\alpha}{1-\alpha}\,\frac{W}{P^d}\,\frac{1}{R^k}\,.$$

Substituting Equation (A.14) into this expression gives:

$$\frac{K}{L} = \frac{\alpha}{1-\alpha}\left[(1+\lambda^p)\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}(R^k)^{\alpha}\right]^{-\frac{1}{1-\alpha}}\frac{1}{R^k} = \left[\frac{\alpha}{1+\lambda^p}\,\frac{1}{R^k}\right]^{\frac{1}{1-\alpha}}$$

and using the expression for R^k from Equation (A.6) leads to the steady-state capital-labour ratio:

$$\frac{K}{L} = \left[\frac{\alpha}{1+\lambda^p}\,\frac{1}{\beta^{-1}-1+\delta}\right]^{\frac{1}{1-\alpha}}\,. \tag{A.15}$$

Substituting the previous expression into Equation (A.13) and re-arranging the terms gives the steady-state consumption in terms of L:

$$C = \left[\frac{\xi}{(1-g)(\beta^{-1}-1)-g\delta}\right]^{-1}\left[\frac{\alpha}{1+\lambda^p}\,\frac{1}{\beta^{-1}-1+\delta}\right]^{\frac{1}{1-\alpha}}L\,. \tag{A.16}$$

Value of L in the steady state remains to be derived. The steady-state form of the optimal labour supply condition (10) combined with Equation (4) is given by:

$$\frac{W}{P^c} = L^{\sigma_l}\big[(1-h)C\big]^{\sigma_c}\,,$$

which can be re-expressed in terms of $\frac{W}{P^d}$ using γ^c as follows:

$$\frac{W}{P^d} = \gamma^c L^{\sigma_l} \big[(1-h)C\big]^{\sigma_c},$$

Combining the previous equation with (A.16) and (A.14) gives the steady-state value of L:

$$L^{\sigma_l+\sigma_c} = \left[\frac{\xi}{(1-g)(\beta^{-1}-1)-g\delta} \cdot \frac{1}{1-h}\right]^{\sigma_c} \left[\frac{\alpha}{1+\lambda^p}\right]^{\frac{1-\sigma_c}{1-\alpha}} \left[\frac{1}{\beta^{-1}-1+\delta}\right]^{\frac{\alpha-\sigma_c}{1-\alpha}} \cdot \frac{1-\alpha}{\alpha}\frac{1}{\gamma^c}. \tag{A.17}$$

The final expression for the steady-state value of C is derived by substituting out L in Equation (A.16) using the previous expression:

$$C = \left[\frac{\xi}{(1-g)(\beta^{-1}-1)-g\delta}\right]^{-\frac{\sigma_l}{\sigma_l+\sigma_c}} \left[\frac{1}{1-h}\right]^{\frac{\sigma_c}{\sigma_l+\sigma_c}} \left[\frac{\alpha}{1+\lambda^p}\right]^{\frac{1+\sigma_l}{(1-\alpha)(\sigma_l+\sigma_c)}} \cdot \left[\frac{1}{\beta^{-1}-1+\delta}\right]^{\frac{\alpha+\sigma_l}{(1-\alpha)(\sigma_l+\sigma_c)}} \left[\frac{1-\alpha}{\alpha}\frac{1}{\gamma^c}\right]^{\frac{1}{\sigma_l+\sigma_c}}.$$

The steady-state value of K follows from Equation (A.15) in a similar manner:

$$K = \left[\frac{\xi}{(1-g)(\beta^{-1}-1)-g\delta} \cdot \frac{1}{1-h}\right]^{\frac{\sigma_c}{\sigma_l+\sigma_c}} \left[\frac{\alpha}{1+\lambda^p}\right]^{\frac{1+\sigma_l}{(1-\alpha)(\sigma_l+\sigma_c)}} \cdot \left[\frac{1}{\beta^{-1}-1+\delta}\right]^{\frac{\alpha+\sigma_l}{(1-\alpha)(\sigma_l+\sigma_c)}} \left[\frac{1-\alpha}{\alpha}\frac{1}{\gamma^c}\right]^{\frac{1}{\sigma_l+\sigma_c}}.$$

The steady-state investment is related to the previous expression via $I = \delta K$:

$$I = \delta\left[\frac{\xi}{(1-g)(\beta^{-1}-1)-g\delta} \cdot \frac{1}{1-h}\right]^{\frac{\sigma_c}{\sigma_l+\sigma_c}} \left[\frac{\alpha}{1+\lambda^p}\right]^{\frac{1+\sigma_l}{(1-\alpha)(\sigma_l+\sigma_c)}} \cdot \left[\frac{1}{\beta^{-1}-1+\delta}\right]^{\frac{\alpha+\sigma_l}{(1-\alpha)(\sigma_l+\sigma_c)}} \left[\frac{1-\alpha}{\alpha}\frac{1}{\gamma^c}\right]^{\frac{1}{\sigma_l+\sigma_c}}.$$

Finally, the steady-state output Y is derived by combining Equation (A.9) with (A.15) and the steady-state value of L given by (A.17):

$$Y = \left[\frac{\xi}{(1-g)(\beta^{-1}-1)-g\delta} \cdot \frac{1}{1-h}\right]^{\frac{\sigma_c}{\sigma_l+\sigma_c}} \left[\frac{\alpha}{1+\lambda^p}\right]^{\frac{1+\sigma_l}{(1-\alpha)(\sigma_l+\sigma_c)}} \cdot \left[\frac{1}{\beta^{-1}-1+\delta}\right]^{\frac{\alpha(\sigma_l+\sigma_c)+\alpha-\sigma_c}{(1-\alpha)(\sigma_l+\sigma_c)}} \left[\frac{1-\alpha}{\alpha}\frac{1}{\gamma^c}\right]^{\frac{1}{\sigma_l+\sigma_c}}.$$

Some important steady-state ratios that follow from the previous expressions and Equation (A.11) are following:

$$\frac{C}{Y} = \frac{1}{\xi}\big[1-g-\frac{\delta}{\beta^{-1}-1+\delta}\big]\,, \ \frac{I}{Y} = \frac{\delta}{\beta^{-1}-1+\delta}\,, \ \frac{X}{Y} = \frac{M}{Y} = \alpha_c\Big(\frac{\gamma^c}{\gamma^m}\Big)^{\eta_c}\frac{C}{Y}\,.$$

References

Adolfson, Malin, Stefan Laséen, Jesper Lindé and Mattias Villani (2007a) Bayesian estimation of an open economy DSGE model with incomplete pass–through. *Journal of International Economics*, vol. 72, pp. 481–511

Adolfson, Malin, Stefan Laséen, Jesper Lindé and Mattias Villani (2007b) Evaluating an estimated new Keynesian small open economy model. *Working Paper Series*, No. 203, Sveriges Riksbank

An, Sungbae and Frank Schorfheide (2007) Bayesian analysis of DSGE models. *Econometric Reviews*, vol. 26, pp. 113–172

Bems R. and Hartelius K. Jönsson (2006) Trade deficits in the Baltic States: How long will the party last? *Review of Economic Dynamics*, vol. 9, pp. 179–209

Benigno P. (2001) Price stability with imperfect financial integration. Mimeo

Brooks S.P. and A. Gelman (1998) General methods for monitoring convergence of iterative simulations. *Journal of Computational and Graphical Statics*, vol. 7, pp. 434–455

Calvo, Guillermo (1983) Staggered prices in a utility maximizing framework. *Journal of Monetary Economics*, vol. 12, pp. 383–398

Canova, Fabio (2008) How much structure in empirical models? *CEPR Discussion Paper*, No. 6791

Canova, Fabio (2007) *Methods For Applied Macroeconomic Research*. Princeton University Press

Chari, V.V., Patrick J. Kehoe and Ellen R. McGrattan (2008) New Keynesian models: Not yet useful for policy analysis. *NBER Working Paper*, No. 14313

Christiano, Lawrence J., Martin Eichenbaum and Charles L. Evans (2005) Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy*, vol. 113, pp. 1–45

Colantoni, Luca (2007) Monetary transmission in Estonia: The interest rate channel, Mimeo

Collard, F. and H. Dellas (2002) Exchange rate systems and macroeconomic stability, *Journal of Monetary Economics*, vol. 49, pp. 571–599

Dabušinskas, Aurelijus and Dmitry Kulikov (2007) New Keynesian phillips curve for Estonia, Latvia and Lithuania. *Working Papers Series*, No. 7/2007, Eesti Pank

Druant, Martine, Slivia Fabiani, Gabor Kezdi, Ana Lamo, Fernando Martins and Roberto Sabbatici (2009) How are firms' wage and prices linked: survey evidence in Europe. *ECB Working Paper Series*, No. 1084

Erceg, Christopher J., Dale W. Henderson and Andrew T. Levin (2000) Optimal monetary policy with staggered wage and price contacts. *Journal of Monetary Economics*, vol. 46, pp. 281–313

Fagan, Gabriel, Jérôme Henry and Ricardo Mestre (2005) An area-wide model (AWM) for the euro area. *Economic Modelling*, vol. 22, pp. 39–59

Fernandez-Villaverde, Jesús and Juan F. Rubio-Ramirez (2007) A baseline DSGE model, Mimeo

Galì, Jordi (1999) Technlogy, employment, and the business cycle: Do technology shocks explain aggregate fluctuations? *The American Economic Review*, vol. 89, pp. 249–271

Gelman A., J. B. Carlin, H. S. Stern and D. B. Rubin (2004) *Bayesian Data Analysis*. Chapman and Hall/CRC

Greenwood J., Z. Hercowitz and G. W. Huffman (1988) Investment, capacity utilization and the real business cycles. *The American Economic Review*, vol. 78, pp. 402–417

Hamilton, James (1994) *Time Series Analysis*, Princeton

Iskrev, Nikolay (2008) How much do we learn from the estimation of DSGE models? A case study of identification issues in a New Keynesian business cycle model. Mimeo

Juillard, Michel (2004) *Dynare Manual*. CEPREMAP

Kattai, Rasmus (2005) EMMA: A quarterly model of the Estonian economy. *Working Papers Series*, No. 12/2005, Eesti Pank

Kollmann, Robert (2001) The exchange rate in a dynamic-optimizing current account model with nominal rigidities: A quantitative investigation. *Journal of International Economics*, vol. 55, pp. 243–262

Lendvai, Julia an Werner Roeger (2008) Estonia 1995 to 2007: Catching-up and beyond. Mimeo

Lundvik P. (1992) *Foreign demand and domestic business cycles: Sweden 1891-1987*. In Business Cycles and Growth, Monograph Series No. 22, Institute for International Economic Studies, Stockholm University

Obstfeld, Maurice and Kenneth Rogoff (2000) The six major puzzles in international macroeconomics: Is there a common cause? *NBER Macroeconomics Annual*, MIT Press.

Poirer, Dale J. (1995) *Intermediate Statistics and Econometrics: A Comparative Approach*, MIT Press

Ratto, Marco, Werner Roeger and Jan in't Veld (2008) Quest III: An estimated DSGE model of the euro area with fiscal and monetary policy. *DG ECFIN Economic Paper*, No. 335

Robert, Christian P. and George Casella (2004) *Monte Carlo Statistical Methods*, Springer

Schmitt-Grohé, Stephanie and Martín Uribe (2003) Closing small open economy models. *Journal of International Economics*, vol. 61, pp. 163–185

Smets, Frank and Raf Wouters (2005) Comparing shocks and frictions in US and euro area business cycles: A Bayesian DSGE approach. *Journal of Applied Econometrics*, vol. 20, pp. 161–183

Smets, Frank and Raf Wouters (2003) An estimated dynamic stochastic general equilibrium model of the euro area. *Journal of the European Economic Association*, vol. 1, pp. 1123–1175

Smets, Frank and Raf Wouters (2002) Openness, imperfect exchange rate pass–through and monetary policy. *Journal of Monetary Economics*, vol. 49, pp. 947–981

Staehr, Karsten (2008) Estimates of employment and welfare effects of personal labour income taxation in a flat–tax country: The case of Estonia. *Working Papers Series*, No. 3/2008, Eesti Pank

Tovar, Camilo E. (2008) DSGE models and central banks. *BIS Working Papers*, No. 258

Uhlig, Harald (1999) A toolkit for analyzing nonlinear dynamic stochastic models easily. In R. Marimón and A. Scott (eds.) *Computational Methods for the Study of Dynamic Economies*, Oxford University Press

Working Papers of Eesti Pank 2009

No 1
Martti Randveer, Tairi Rõõm
The Structure of Migration in Estonia: Survey-Based Evidence

No 2
Andrew Hughes Hallett, Rasmus Kattai and John Lewis
Can We Rely on Real Time Figures for Cyclically Adjusted Budget Balances?

No 3
Jan Babecký, Philip Du Caju, Theodora Kosma, Martina Lawless, Julián Messina and Tairi Rõõm
Downward Nominal and Real Wage Rigidity: Survey Evidence from European Firms

No 4
Jan Babecký, Philip Du Caju, Theodora Kosma, Martina Lawless, Julián Messina and Tairi Rõõm
The Margins of Labour Cost Adjustment: Survey Evidence from European Firms