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Credit Risk Model for the Estonian Banking Sector

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Abstract

This paper gives an overview of the credit risk model that has been developed for the Estonian banking system. The non-performing loans and loan loss provisions of the four largest banks and the rest of the banking sector have been modelled conditional on the underlying economic conditions: economic growth, unemployment, interest rates, inflation, indebtedness and credit growth. The model highlights the importance of economic growth as the most influential factor behind the soundness of the banking sector in the latest downturn. The expected fall in output volatility will probably decrease the relative importance of output growth and increase the role of interest rates in the future.

JEL Code: E32, E37, G17, G21

Keywords: credit risk, stress testing, financial soundness indicators, Estonian banking sector

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Non-technical summary

The model developed in the paper serves as a tool for analysing and stress testing the financial stability of the Estonian banking system by assessing its openness to credit risk, hence the name The Credit Risk Model. It is used regularly, twice a year, in the preparation of the Financial Stability Report of Eesti Pank and also as part of the IMF Financial Sector Assessment Program (FSAP).

As in most countries, credit risk clearly outweighs the other risks in the Estonian banking sector, as banks' loans to companies and households constitute about 80% of the sector's assets. Therefore it is the ability of households and companies to cope with unexpected negative shocks that largely determines the sustainability of the whole banking system, while also highlighting banks' responsibility to create sufficient capital buffers. Shocks that may lead to a household being unable to fulfil its contractual agreements may include the loss of a job, a fall in salary, a rise in interest rates, and other such events. Companies may face similar difficulties after a substantial drop in demand for their products and a consequent fall in revenues, and also as a result of increased loan servicing costs and similar changes.

The inability of households and companies to meet their contractual agreements is quantified by estimating the probability of them defaulting in each period. In doing this, the model distinguishes between consumption credit and mortgage clients. These client groups are believed to react somewhat differently in finding ways to service their loans in times of distress, as this paper also finds. It holds even if a household has taken both types of credit, and this is not restricted by the model. The probable reason is that credit holders are more cautious when handling financial obligations related to their residential property, their home.

The probability of a credit client defaulting, whether a household or a company, is conditional on the underlying macroeconomic environment, which in the model is characterized by GDP growth, the unemployment rate, interest rates and inflation. These variables carry information on probable external negative shocks at the aggregate level. Other factors that influence the probability of default are the indebtedness of credit clients and overall credit growth. Indebtedness shows financial deepening and the obligations to income ratio, an increase in which makes it more difficult to cope with shocks if they occur. Overall credit growth conveys information on general conditions in the credit market, showing how easy it is to get access to credit and roll earlier contracts over if that should be necessary in order to avoid default.

The credit risk model separates the four major banks and the rest of the sector, which is a sufficient level of disaggregation, given the high concentration within the sector, where the total market share of the four biggest is approximately 94%. In order to get a more structured view of the loss provisioning process and to understand bank-specific nuances, the model disentangles non-performing loans, i.e. loans that are more than 60 days past due, and loan loss provisions for each bank and client group. Losses are provisioned on the basis of the non-performing loans and the loss given default ratio, which broadly resembles the coverage of the collateral. Provisioning procedures are highly discretionary and differ across the banks, and also within a particular bank from period to period. Therefore the model aims to find the average expected provisions instead of predicting each bank's provisioning decisions for every period.

Non-performing loans, on the other hand, are more tightly linked to the economic environment and less to discretion, and can be explained by the model with a higher precision. Past due loans are modeled as a dynamic process, in which the outstanding stock is calculated from its own value in the previous period, the new additional non-performing loans from the new defaulted clients, and previously non-performing loans that have started performing again.

The model is suitable for three types of analysis, also discussed in the paper: sensitivity analysis, deterministic and stochastic scenario analysis. Sensitivity analysis assesses each risk factor's impact in isolation from the others and gives an insight into which factors are the most influential. Sensitivity analysis with the current model shows the high importance of economic growth and its fluctuations, as a proxy for households' disposable income and companies' revenues. Although interest rates have not been a great source of unexpected shock, in contrast to the economic growth rate, they are highly influential as they affect debt servicing costs in the most direct way. However, the full impact of changes in the Euribor, the base interest rate in most contracts, takes about half a year to pass through. This is the time it takes to re-price all the contracts fixed to the 6-month Euribor.

In the paper loan loss provisions are conditioned on two scenarios of macroeconomic development to produce future projections. The two scenarios, Base and Negative Risk, are taken from the most recent Eesti Pank official forecast, released in Autumn 2009. Both scenarios expect risks to fall from early 2010 until the end of the forecast horizon at the end of 2011. This is reflected in the steadily falling non-performing loan and provisions rates. The same conclusion can be drawn from the stochastic simulation exercise, which matches the official forecast scenario-based outcomes fairly well. However, simulations also show that it takes a long time for the distress to vanish compared to how quickly it emerged.

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1. Introduction

The beginning of systematic stress testing dates back to the early 1990s, when banking supervisors and regulators sanctioned it as an important component of market risk monitoring (Blaschke et al., 2001). Stress tests are widely considered to play a central role in financial stability monitoring and in avoiding crises. There are serious consequences to be avoided — research has shown that output losses resulting from a financial crisis are about 9% on average (Reinhart and Rogoff, 2009), which is substantially larger than the losses caused by non-bank crises (Haugh et al., 2009). The latest World Economic Outlook (2009) reports a historical average loss of 10% but also emphasises substantial variations between countries, as the middle 50% of crisis episodes caused losses ranging between -26% and $+6\%$. [1] In this light the importance of the regular monitoring of risks is more than clear. Thorough risk analysis may give early warning signals, indicating vulnerability in the financial sector, and encouraging the regulatory body to take precautionary actions to avoid the crisis. [2]

Models for performing stress tests vary greatly between institutions and the choice of modelling framework is often predetermined by the availability of data. Individual financial institutions operating in the market are closer to the relevant data and are better able to analyse their own credit risk than is a supervisory body, and therefore the choice of tools is larger for an individual institution than for a supervisory agency or a central bank. Nevertheless, it is important for a supervisory body and/or a central bank to have a tool for evaluating potential problems that may occur in severe economic conditions. The focus in this case is not only on individual banks but on the banking system as a whole (Basel Committee on Banking Supervision, 2000a), ans on top of this the Basel Committee on Banking Supervision (2000b) sees the role of a supervisory agency as requiring that banks have an effective system in place to identify, monitor and control this risk.

A leading role in developing appropriate tools at central banks has been played by the IMF. In May 1999 the IMF, in co-operation with the World Bank, instituted the Financial Stability Assessment Program (FSAP) to promote soundness in the financial systems in member countries (Blaschke et al., 2001). The ultimate goal of the programme is to reduce the number of crises

[1] In comparison, the European Commission (2009) predicts a -3.7% decline in potential GDP in the Euro Area and -5.9% in EU8, as a direct result of the latest global financial crisis.

[2] Early warning is an important function of the monitoring and stress testing procedure. Frydl (1999) shows that the materialised loss depends on the speed with which regulatory bodies resolve the crisis. The speed of action depends heavily on how quickly the authorities are warned about possible problems and the proportion of losses that can still be avoided.

worldwide by investigating the weaknesses of each country's financial system and suggesting remedial policies (Kalirai and Scheicher, 2002). As a positive consequence, many authorities have introduced or developed further the practice of stress testing as a part of the FSAP (Foglia, 2009).

The model developed in the present paper concentrates on credit risk, which is defined by the Basel Committee on Banking Supervision (2000b) as the risk that a bank borrower will fail to meet its contractual obligations as they have been agreed. Credit risk is by far the most important risk in the Estonian financial system as loans constitute about 80% of the Estonian banking sector's assets. [3] Credit risk models of various types typically use either non-performing loans (NPL) or loan loss provisions (LLP) as a measure of distress. The framework introduced in the presenr paper is richer in structure and binds both measures into a system. Non-performing loans are generated by the default rates of companies and households, which in turn are driven by the underlying macro fundamentals which shape the income of banks' clients and the costliness of servicing loans. Loan loss provisions depend on non-performing loans as an accounting measure, and on the expected loss ratio.

Stress tests carried out with the model include sensitivity and scenario analysis. Sensitivity analysis provides information on how non-performing loans and the expected loan losses respond to shifts in one or several underlying macro indicators such as economic growth or interest rates. Scenario analysis, which is more complex, mimics the consistent movement of all the macro variables in a particular economic conjuncture. In the present paper two macro scenario generating strategies are explored. The first method takes alternative scenarios from Eesti Pank's official forecast from Autumn 2009, while in the second experiment a vector auto-regression (VAR) model is estimated to produce stochastic paths of macro fundamentals and financial variables.

Both experiments convey the same message: risks will be lower from early 2010 and distress measures should start to decrease from then onwards. Nevertheless, the fall in the NPL and LLP ratios is not as quick as their rise in 2008 and 2009 was. The normalisation process is actually considerably slower and the pre-crisis levels remain out of reach by the end of the forecast horizon in December 2011.

The contribution of the present paper to the literature is threefold. Firstly, it introduces the dynamic equation specification of non-performing loans to extract missing data on the probability of default rates. This approach becomes useful when no official data on default probabilities exist. Secondly, the model

[3] The other most common risks are market, interest rate, exchange rate and liquidity risks. These are less likely to cause the failure of a financial institution in Estonia. However, liquidity risk increased noticeably in 2008.

allows new additional non-performing loans and loans that are carried over or have become performing again to be separated. The same applies to loan loss provisions. Thirdly, the system of macro indicators, non-performing loans and loan loss provisions is set up to give a consistent view of two distress measures than choosing only one of them as common in the literature.

The paper is divided as follows: Section 2 gives an overview of the related literature and the practices in various institutions. Section 3 presents the modelling strategy and the structure of the credit risk model. It gives an overview of how the initial impulses in the macro variables are finally transmitted into loan loss provisions through a chain of processes. Section 4 gives a number of empirical examples from running the credit risk model and comments on the test outputs. Section 5 draws conclusions on the paper and gives some ideas for future work on potential improvements to the current version of the model.

2. Related literature and practices

The definitions and meanings of stress test, scenario analysis and sensitivity analysis vary in the literature. Throughout this paper the definitions of the Bank for International Settlements (BIS) are used (Committee on the Global Financial System, 2001). According to the BIS, stress testing is a generic term to describe various techniques for exploring the potential vulnerability of financial institutions to exceptional but plausible events. Stress tests can be either sensitivity tests or scenario analysis.

The sensitivity test is a univariate approach for assessing one risk factor's impact on the financial data. Analysing one shock in isolation has its advantages and disadvantages. The strengths of the sensitivity analysis are (a) it conveys important information on the performance of the model itself and (b) it brings out the most important factors that drive clients into insolvency. The most crucial drawback of the approach is that it ignores the simultaneity or interdependence of risk factors.

Scenario analysis is a more complicated way of exploring the risks. It provides an integrated view of economic fundamentals and financial data, as risk factors are projected to evolve in a consistent manner. Due to its multivariate nature, scenario analysis is generally believed to be more realistic than sensitivity tests, since in reality all the risk factors interact (van den End et al., 2006). Literature on this subject distinguishes between two types of scenario: historical and hypothetical. Historical scenarios draw their financial data from macro episodes that have already occurred, whereas hypothetical analysis tries to see what happens in circumstances that have never occurred before (Hadad et al., 2007). Hypothetical scenarios are more flexible because they are not restricted in formulating potential events (Blaschke et al., 2001). It may be difficult to justify a hypothetical scenario without any historical comparison, but they are realistic in the sense that new shocks may have nothing in common with what has been experienced in the past. Hypothetical scenario analysis becomes the only option when structural breaks in the financial system (deregulation, consolidation, change of currency) have annulled the information content of past episodes (Quagliariello, 2009). [4]

There is a vast wealth of literature on how stress scenarios should be produced and put into the stress-testing model (see for example Basel Committee on Banking Supervision (2009) and Committee on the Global Financial System (2001)). Foglia (2009) provides a comprehensive overview of the practices of central banks and supervisory authorities. In principle, stress scenarios can be translated into the macro environment using a structural econometric model, a VAR model or pure statistical methods. Most of the thirteen institutions covered by the study used macroeconomic models developed for monetary policy analysis and forecasting. The strength of this approach is a coherent prediction of the macro variables, responding to the shocks added to the model. A concern that may arise is whether the macro econometric model responds adequately to the shocks imposed, given its structure, and its coverage of the variables and the linkages between them. [5] Missing links would underestimate the severity of a scenario and threats to the financial system. In a similar vein Foglia (2009) raises the issue that linear macro models fail to produce a consistent relationship between the variables, which may become nonlinear at times of stress.

VAR or vector error correction models (VECMs) are used if a macro economic model is not available or suitable for generating the desired scenarios (Foglia, 2009). VARs and VECMs are widely used alternatives, appreciated for their flexibility and relatively low building costs. The drawback of these models is that they are to a large extent statistical, which means that outcomes are difficult or sometimes even impossible to interpret. Despite the problems of explaining the outcome, they still provide coherent shock scenarios.

Depending on the scenario chosen, stress tests can be carried out using either top-down or bottom-up principles. Authors are more consistent in defin-

[4] Matz (2007) summarises that sometimes the term "scenario" is used to indicate the deterministic path of the underlying macro variables, while "stress test" is considered to indicate their probabilistic paths (and therefore also the values for the balance sheet items). In other cases "stress test" refers to a univariate and "scenario" to a multivariate analysis. According to the third set of definitions the term "sensitivity test" is used to distinguish univariate analysis from the multivariate analysis labelled as "stress test".

[5] For example, standard macro models are not particularly good at mimicking oil price shocks, unless they are specially designed to do so. The comparison of the Euro Area central banks' models by Fagan and Morgan (2005) proves that shortcoming.

ing the top-down approach, which means modelling aggregated financial data. The meaning of the bottom-up method differs somewhat in the literature. According to Čihák (2007) the bottom-up approach only refers to the level of disaggregation, as it separates individual portfolios and the analysis can be done in one centre, such as the central bank or a supervisory agency. Hadad et al. (2007) also emphasise the separate roles of supervisors and financial institutions in the bottom-up approach, where the supervisory agency defines the shock to be analysed and collects the impact evaluations carried out by individual institutions, and then aggregates them. Therefore the bottom-up approach is sometimes also called an aggregate test, in which a central co-ordinator has an important role (Committee on the Global Financial System, 2000).

Sorge and Virolainen (2006) make a further distinction within the group of top-down models by splitting them into balance sheet models and value-at-risk models (VaR). Balance sheet models explore the links between financial institutions' accounting items such as non-performing loans, loan loss provisions or write-offs, and the business cycle. These models are relatively simple and typically linear, using the most relevant macro variables, such as GDP, inflation, and interest rates, to account for variations in the accounting items. The impact of macro fundamentals either can be explored by direct estimation of reduced form relationships by applying time series or panel estimation, or can be drawn from a fully fledged macro model. Sorge and Virolainen (2006) emphasise intuitiveness, low computational burden and broader characterisation as advantages of the balance-sheet models. On the other hand there are also several drawbacks, including linear relationships that may be too simplistic, parameter instability, and the lack of feedback effects, and it is also true that loan loss provisions and non-performing loans could be too noisy as indicators of credit risk.

VaR models also use multiple macro economic variables to gauge their effects on financial institutions' portfolios but the emphasis is on probabilistic outcome. The vulnerability of the financial system is captured by a probability distribution of losses based on a suitable macro scenario. This method avoids the possible shortcomings of the balance sheet models but has other concerns, as value-at-risk measures are not additive across portfolios, and so most models in this strain focus on aggregate portfolios (Sorge and Virolainen, 2006). The shortcoming may become quite restrictive at times of high risk because it does not permit analysis of an individual institution's performance nor the possible contagion effects.

Čihák (2007) provides yet another categorisation of models, distinguishing between loan performance models and default rate models. The first class of models is similar to what was earlier described as balance sheet models, using non-performing loans, loan loss provisions or default frequencies as distress measures, as determined by macroeconomic conditions. These models are not necessarily related to the top-down approach and vary greatly in their level of aggregation — some focus on aggregate data, others work at industry level, but banks can also be separated out if the data is available.

The second class of models are built on client-specific micro data, which are explained by macro economic conditions. Macro data can enter the default rate model directly, or another satellite model could be used first to draw a link between macro data and an individual borrower's performance characteristics such as future income. Foglia (2009) argues that default rate models, i.e. methods that use financial data to forecast bankruptcies, may detect possible problems in loan portfolios sooner than models based on loan classification data such as non-performing loans or loan loss provisions. However, these models require large datasets that are available only to selected institutions.

Stress tests are most often performed in the spirit of extreme value theory, maximum loss approach (also known as worst case scenario analysis) or contagion analysis. Extreme value theory, as its name suggests, deals with extreme events in financial markets. Rather than the distribution of all returns, it concentrates on the distribution of extreme returns, which are considered to be independent over a long time period (Longin, 1999). The maximum loss approach finds the combination of market moves that would cause the greatest loss to the portfolio (Committee on the Global Financial System, 2000). Contagion analysis quantifies the transmission of one financial institution's failure to others and the possible impact on the whole financial system.

Moretti et al. (2008) give an overview of the different types of stress test used in the IMF's FSAP and show that contagion analysis has become more common over the years. The increased popularity of the method stems from the tightened linkages between financial institutions within each country and across borders. Alessandri et al. (2009) refer to an unpublished study by Gai and Kapadia (*mimeo*, Bank of England) which finds that the effect of the greater connectivity of financial networks is twofold. Firstly, it enhances risk sharing and therefore lowers the likelihood of crises. But secondly, if a crisis does occur, the impact of it would also be more severe. This effect is reinforced by financial innovations and general macroeconomic stability (Gai et al., 2008).

Regardless of the modelling technique or stress test approach that is applied, no results represent a final truth. Bunn et al. (2005) state that "... no single model is ever likely to capture fully the diverse channels through which shocks may affect the financial system. Stress testing models will, therefore, remain a complement to rather than a substitute for, broader macroprudential analysis of potential threats to financial stability." Any model built on economic data is a simplification of reality and unable to take into account all the complex structures and sources of shocks as they occur in the real world.

3. Structure of the Credit Risk Model

The credit risk model is built for the whole banking sector in Estonia. The model should give an insight into the ways how economic conditions may affect banks' clients ability to service their obligations and how this is reflected in banks' portfolios. The aim is to use simultaneously both the most common measures of distress, non-performing loans and loan loss provisions, with the second as a function of the first. This is done in order to gain a more comprehensive overview of the health of the banks' portfolios. Concentrating on only one of these measures would give only partial information whereas in reality they are interlinked.

Non-performing loans and loan loss provisions are linked by structural equations on a monthly basis. Modelling both of them together is also motivated by the fact that in practice it is easier to correlate non-performing loans with the underlying economic conditions but the main interest lies in the potential losses of the banks (LLPs). The structure of the model does not include banks' capital buffers or profits. Losses from credit exposures have to be compared against banks' capital reserves externally, outside the credit risk model, in order to establish which set of circumstances may run a bank into insolvency.

Four major banks — Danske Bank (d), Nordea (n), SEB (s) and Swedbank (w) and enter the model individually, while the the others are grouped together (o). [6] Each bank's credit portfolio is divided into three sectors. These are mortgage (h), corporate (f) and consumer credit (c). [7]

Non-performing loans, N, are modeled for sector $m \in \{c, f, h\}$ of each bank, $b \in \{d, n, o, s, w\}$. Equation 1 generalises the dynamic process that generates the outstanding stock of non-performing loans. The first component on the right hand side of the equation stands for the *new* non-performing loans, which are drawn from a bank's exposure, E, at a probability of default rate, Π. Π stands for the probability of default *at the given moment* by reflecting

[6] There are 7 subsidiaries and 11 branches operating in Estonia. The current categorisation is optimal because the 14 smallest banks' share of the credit market is only about 6%. Further distinguishing between them would add too little to the analytical power of the model.

[7] Foglia (2009) emphasises that central banks and/or supervision authorities should concentrate on bank level performance rather than an aggregate system-wide portfolio. Aggregate data may conceal substantial variation at portfolio or bank level, leaving the potential for failure of individual institutions undiscovered.

the rate at which previously solvent clients become insolvent. It characterises new draws from the pool of solvent contracts in contrast to the alternative definition, where PD stands for the share of insolvent contracts in the portfolio. In other words, the first component measures new *additional* non-performing loans that accumulate until they are either written off or the client has become solvent again. Parameters η and ψ are used to differentiate banks by allowing their clients' to react differently to changes in the average PD of sector m. [8]

$$N_t^{b,m} = \left(\eta^{b,m} + \psi^{b,m}\Pi_{t-k}^m\right) E_{t-k}^{b,m} + \left(1 - \rho^{b,m}\right) N_{t-1}^{b,m} - M_{t-1}^{b,m} + \zeta_t^{b,m}. \quad (1)$$

The second component stands for the "healed" contracts, loans that were non-performing in the previous period but have become operating again. This may be due to the client restructuring or refinancing the loan, selling the collateral and paying back the loan or becoming solvent again. [9] In the current set-up of the model the rate of recycling, ρ, is constant ($0 \leq \rho^{b,m} \leq 1$). Constancy of ρ is quite a bold assumption. In principle ρ could be cycle-sensitive and affected by the same factors as PD. However, attempts so far at modelling ρ as a time dependent variable have not yielded promising results.

The second component is relevant to replicate the decrease in the observed non-performing loans (occasions when $\Delta N_t < 0$). New non-performing loans, $\left(\eta^{b,m} + \psi^{b,m}\Pi_{t-k}^m\right) E_{t-k}^{b,m}$, are always non-negative because $\Pi \geq 0$, $E \geq 0$, $\eta \approx 0$ and $\psi \geq 0$. In principle, the fall in the non-performing loans could be due to loans being written-off, M, which stems from the principle that a non-performing loan is removed from the balance sheet once the bank writes it off. But equation 1 is shaped to be able to explain improvements in banks' portfolios even without write-offs, i.e. when $M = 0$. This is especially plausible in the economic recovery phase, when banks' clients are more likely to become solvent again.

Foglia (2009) calls a non-performing loan a "retrospective indicator" of asset quality, which should be controlled by using lagged values of the probability of default. This is represented by k in equation 1. The value of k is related to the definition of a non-performing loan, i.e. how long the loan has to

[8]Equation 1 simplifies the process by defining the mass of contracts where the new non-performing loans can be drawn from as E, which actually contains the already outstanding non-performing loans. Using $E - N$ would be more correct but is left aside for the sake of later modifications of the equation and the ease of interpretation. Parameter estimations would change only marginally if $E - N$ were used instead of E.

[9]One aspect related to restructuring of a loan is that technically it becomes operating and will not show up as a non-performing loan any more, but the quality of the bank's portfolio may decrease as a result. Exploring this aspect, however, remains beyond the scope of the current paper.

be past due before it is considered non-performing. In this paper loans 60 days past due are treated as non-performing, which means that in fact the clients became insolvent two months ago and N_t is actually generated by PD at $t-2$ from E_{t-2}.

Dividing both sides of equation 1 by exposure, E_{t-2}, transforms the left hand side of it into a non-performing loan ratio (assuming that N_t was in fact already non-performing two periods before, i.e. at the lag of the denominator). The equation to be empirically estimated becomes:

$$\frac{N_t^{b,m}}{E_{t-2}^{b,m}} = \eta^{b,m} + \psi^{b,m}\Pi_{t-2}^{m} + \left(1-\rho^{b,m}\right)\frac{N_{t-1}^{b,m}}{E_{t-2}^{b,m}} + \omega_t^{b,m}. \tag{2}$$

Equation 2 disentangles PD and the NPL ratio although Blaschke et al. (2001) show that the NPL ratio itself can be treated as a default frequency measure. They say that this is true if the assumption of the normal distribution of individual exposures holds and there is no time variance in the recovery rate (loss given default, LGD). Further in the paper a time variant recovery rate is introduced and therefore the concepts of the NPL ratio and PD are kept different.

Write-offs, M, do not appear in equation 2 because loans have been written off only exceptionally in Estonia. With almost zero variation it has no explanatory power and is therefore left out of the equation. Any reduction in N can only be attributed to the "recycling" of the loan, supporting the dynamic set-up of the equation. Error term ω has the property of $\omega^{b,m} \sim N(0, \sigma_{\omega^{b,m}})$ and relates to ζ by $\omega_t^{b,m} = \zeta_t^{b,m}/E_{t-2}^{b,m}$.

Since there are no official data available for PD it has been stripped out from the related data. This is done by estimating equation 2 sector-wise, by aggregating the data for one sector from all the banks. The probability of default of a sector m, Π^m, is proxied by a set of macro indicators, X, and the corresponding set of parameters, β^m:

$$\frac{N_t^{m}}{E_{t-2}^{m}} = \frac{e^{f(\beta^m,X)}}{1+e^{f(\beta^m,X)}} + (1-\rho^m)\frac{N_{t-1}^{m}}{E_{t-2}^{m}} + \xi_t, \tag{3}$$

where N^m and E^m equal $\sum_b N^{b,m}$ and $\sum_b E^{b,m}$ respectively. Equation 3 regresses the NPLs directly on macro fundamentals, and the combination of them generates the PD series because $e^{f(\beta^m,X)}/(1+e^{f(\beta^m,X)}) = \Pi^m$ as imposed by the initial equation 1. Note that parameters η and ψ do not appear in the equation since it extracts the *average* PD within the sector across the banks. [10] Logistic form is used to keep the proxy of the probability of default always greater than zero. Another reason for using the logistic form is to add non-linearity to the system. This is in line with the finding that the financial system is hit more severely on occasions when shocks are large and the relevant macro variables are further away from their fundamentals (Foglia, 2009).

The list of macro variables used in different studies is quite long. The core list includes such variables as GDP growth, real and nominal interest rates, inflation and indebtedness. Hadad et al. (2007) also use money aggregates M1 and M2 and the stock price index and Marcucci and Quagliariello (2008) include exchange rate and output gap data. The oil price was found to be a significant risk factor in a study by Simons and Rowles (2009), while Kalirai and Scheicher (2002) extend the list by including consumption spending, employee compensation and new car registrations.

Howard (2009) argues that the choice of variables should be based on stress scenarios and the types of risks that will be analysed with the model. Scenarios for the developed credit risk model are produced using Eesti Pank's forecast model, and so the macro indicators used in the function of PD are those which also appear in the macro model — unemployment, real output growth, nominal interest rates and inflation, the last two are used to calculate the real interest rate on stock. These coincide with the conventional list of variables used in similar models, and they also meet the key requirement of economic plausibility, stressed by Foglia (2009), which states that all predictive variables must have clear meaning and an interpretable relationship with the dependent variable.

In the credit risk model cyclical pressures are captured by the unemployment rate and economic growth. [11] The output gap has been left out of consideration, although Marcucci and Quagliariello (2008) find it to be the most powerful cyclical indicator. Problems may arise with the predictability of the output gap at stressful times, when potential output may, for various reasons, shift downwards (as discussed in the introduction). These permanent or temporary structural shifts can only be detected *ex post* and remain questionable *ex ante*, making them a major source of error. The economic growth rate is therefore considered to be a more reliable indicator of the economic cycle.

[10] A small exception is allowed in equation 2 when it is used to extract the PD of the consumption credit sector. The performance of the NPL ratio of the "other banks" differs greatly from the average for the sector and for that reason other banks' nonperforming loans are not included in the sector totals — for $\sum_b N_t^{b,c}$ and $\sum_b E_t^{b,c}$ the set of banks is smaller, $b \in \{d, n, s, w\}$.

[11] The correlation between unemployment and economic growth is not statistically significant, leaving aside multicollinearity issues.

Both cyclical indicators are normalised to have zero mean and unit variation. [12] The normalised unemployment rate, $\tilde{u}_t$, is given by $(u_t - \overline{u})/\sigma(u)$ and normalised output growth, $\tilde{y}_t$, is given by $(y_t - \overline{y})/\sigma(y)$, where $(\bar{.})$ denotes mean value and $\sigma(.)$ denotes the standard deviation of a variable. Unemployment is expected to have a greater impact on households' PD, as it implies a sudden stop in the stream of income and leads to households being unable to service their obligations. Output growth is expected to reflect the performance of companies more closely by illuminating the ease with which they can sell their products and get revenues to service their debts. On the other hand, output growth is a close proxy for households income as well, as it is highly correlated with disposable income.

Real interest rates have a clear impact on borrowers' ability to service debt. However, Evans et al. (2000) claim that their relevance decreases with high economic growth rates. The real interest rate on the stock of loans, r^s, is defined as the difference between nominal interest rates on stock, i^s, and the weighted inflation rate: $r_t^s = i_t^s - [\tau_m \pi_t + (1 - \tau_m) \mathsf{E}\{\pi\}]$. Parameter τ $(0 \leq \tau \leq 1)$ shows how much weight the agents put on the current inflation rate, while $1 - \tau$ shows the importance of inflation expectations $\mathsf{E}\{\pi\}$, which are proxied by the historical mean, $\overline{\pi}$. For the PD function the real interest rate is normalised to have zero mean and unit variance: $\tilde{r}_t^{s,m} = (r_t^{s,m} - \overline{r}^{s,m})/\sigma(r^{s,m})$.

PD is also considered to be a function of the debt to output ratio in the sector (indebtedness), $e_t^m = E_t^m / Y_t P_t$, where Y is real GDP and P is the price level. Debt ratio is also normalised for the sake of comparison with the other regressors $\tilde{e}_t^m = (e_t^m - \overline{e}^m)/\sigma(e^m)$. Indebtedness is added to the equation in order to account for the increase in monthly obligations (principal and interest payments) compared to earnings, which makes a client more likely to stop servicing a loan if an unfavourable shock hits, *ceteris paribus*.

The last determinant of PD is the normalised annual credit growth, $\tilde{g}_t^m = (g_t^m - \overline{g}^m)/\sigma(g^m)$, where $g_t^m = \Delta_{12} E_t^m / E_{t-12}^m$. The argument here is that at times of rapid credit growth it is easier to get access to refinancing for existing debts and thus avoid the contracts becoming non-performing. A somewhat more loosely related reason lies in the correlation of asset price movements and credit growth. A boom in a credit market increases asset values and allows further borrowing against the increased value of the collateral. This increased collateral should enable a client in default to sell an asset and pay back all the debt, but this becomes problematic at a time when the asset value has fallen back. Combining all the indicators the function to extract the average PDs of mortgage and consumption credit clients and of companies becomes:

[12] The frequency of the originally quarterly GDP growth and unemployment rate data is increased by matching the quadratic average.

$$f(\beta^m, X) = -\beta_0^m + \beta_1^m \tilde{u}_{t-l} - \beta_2^m \tilde{y}_{t-o} + \beta_3^m \tilde{r}_{t-j}^{s,m} + \beta_4^m \tilde{e}_t^m - \beta_5^m \tilde{g}_t^m, \quad (4)$$

where l, o and j denote the time lags which are required for the unemployment rate, output growth and real interest rate to affect PD. The choice of lags is based on the statistical significance of the estimated parameters. Longer time lags would imply greater savings or other resources which can be used to service the debt during the period when earnings have fallen or the price of credit has increased.

The estimation results for equation 3 are presented in Table 1. Unemployment appears to be a relevant factor in the PD of consumption credit and mortgage clients, while it remains unimportant for companies (although it has the right sign). Economic growth has a greater impact on payment performance of companies, but also remains a significant factor for households. The interest rate matters most for the mortgage sector. It can be explained by the substantial size of individual loans and high monthly service costs at relatively low interest rate levels. These mean that any change in the interest rate has a major impact on clients' ability to service their loans. The weight of the contemporaneous inflation rate is zero or close to zero for all the sectors, suggesting that companies and households rely on the expected inflation rate when calculating the real interest rate. Indebtedness is relevant in all three sectors, and is at almost the same level. Recycling rates are also very similar, varying between 0.32 and 0.39.

The selection of lags shows that it takes longer for mortgage clients to be affected by the unfavorable economic conditions. Consumption credit clients and companies react more quickly in this respect. This is especially pronounced in the case of economic growth, which translates into wage and disposable income growth for consumption credit clients.

The number of observations used to estimate the equations ($nob = 73$) is relatively high because monthly data is used instead of quarterly as is most usual. However, the relationship between the NPL ratio and the selected macro indicators may be influenced by the lack of a full cycle in the data, as discussed by Simons and Rowles (2009). The common sample only starts in August 2002 and therefore the macro indicators only capture three quarters of the last cycle. Balance sheet data, on the other hand, covers all the possible states of the credit market, from the low initial indebtedness of households and companies, through rapid credit growth hand in hand with the asset price boom, and finally up to the collapse of the demand for credit. [13] Having the data from the period

[13] Considering that indebtedness prior to 2002 was almost negligible, the properties of the model could not be improved by extending the sample backwards anyway.

of the credit demand collapse as well somewhat answers the criticism brought up in the Basel Committee on Banking Supervision (2009) that models built on the data of a stable period were not able to pick up severe shocks of the type that recently occurred, and hence they underestimated the vulnerability within the financial system.

Table 1: Estimation results: Probability of default

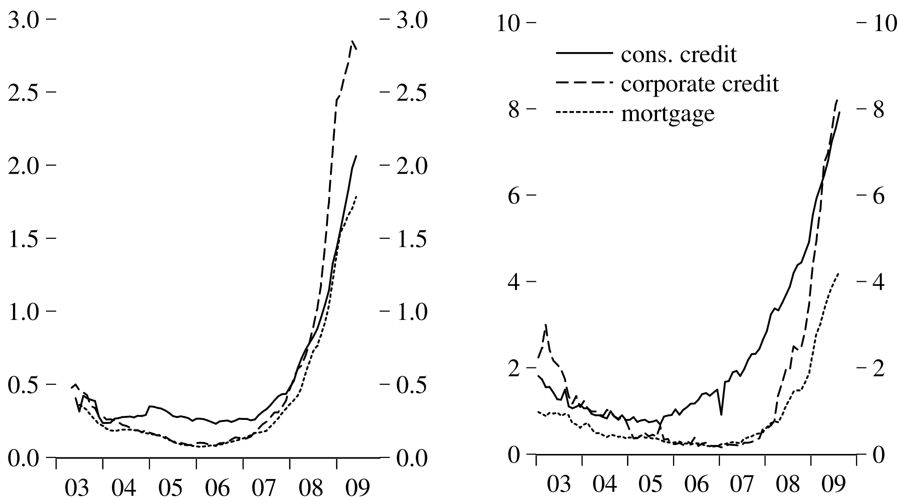
Param.	Descrption	Mortgage	Consumption	Companies
$\hat{\beta}_0$	intercept	6.06^{***}	5.63^{***}	5.63^{***}
$\hat{\beta}_1$	unemployment rate	0.15^{***}	0.05^{**}	0.07
$\hat{\beta}_2$	economic growth	0.11^{**}	0.14^{***}	0.22^{**}
$\hat{\beta}_3$	real interest rate	1.18^{***}	0.26^{**}	0.53^{**}
$\hat{\beta}_4$	indebtedness	0.58^{**}	0.47^{**}	0.48^{*}
$\hat{\beta}_5$	credit growth	0.45^{**}	0.53^{***}	0.57^{**}
$\hat{\tau}$	inflation weight	0.05^{**}	0.00	0.01
$\hat{\rho}$	recycling rate	0.39^{***}	0.34^{***}	0.32^{***}
l	lag of unempl. rate	5	4	4
o	lag of economic growth	4	0	0
j	lag of interest rate	4	3	2
R^2		0.99	0.99	0.99

Notes: Estimation period: August 2003 – August 2009 ($NOB = 73$). ***, ** and * denote statistical significance at levels of 1%, 5% and 10% respectively.

All the estimated equations exhibit extremely high fit ratios, which may look alarming at first glance. However, the reason for this lies in the specification of the equation, which includes an autoregressive component with great explanatory power for a fairly persistent process.

Figure 1 depicts the extracted PD rates and NPL ratios in parallel. Consumption credit and mortgage clients share very similar PD rates, which suggests that either the two groups are not different in their characteristics and/or most households are tied to both types of credit. NPL ratios, on the other hand, exhibit clear differences, as they are considerably lower for mortgage clients. Technically, the spread of the NPL ratios originates from the different dynamic properties of the NPL equations. This is well in line with the perception that whereas households are more careful with servicing debts related to their habitation, it is less problematic for them to have difficulties with consumption credit.

The corporate sector's PD was comparable with that of the households, especially in the beginning of the period. There was some divergence from mid 2008, when it started to rise faster. Figure 1 also illustrates that PDs differ from NPL ratios both in magnitude and dynamics, reinforcing the argument that the NPL ratio is not a suitable proxy for the PD in the current framework.



(a) Probability of default rates.

(b) Non-performing loan rates.

Figure 1: Sectoral break-down of PD and NPL rates.

In the following step the NPL equations are estimated for individual banks and sectors, employing the structure presented in equation 2 and making use of the extracted PD series. Summary of the results is presented in Table 2 (individual equations are not shown due to data confidentiality). The table shows that the results for η equal zero in almost all equations. The positive η value in one case reflects that the NPL ratio was high even when the average PD in the sector was very low.

Parameter ψ illustrates quite extensive variations in the client base of the banks, ranging between 0.22 and 2.84 (with a standard deviation of 0.67 across the equations). If $\psi < 1$ then the clients are less vulnerable to changes in the macro environment compared to the sector average and if $\psi < 1$ the opposite is true.

Table 2: Summary of estimation results: Non-performing loans

	Min.	Max.	Mean†	p^* $[0, 0.01)$	$[0.01, 0.05)$	$[0.05, 0.1)$	$[0.1, \infty)$
				mortgage			
$\hat{\eta}$	-0.00	0.00	0.00	1	1	0	3
$\hat{\psi}$	0.33	2.19	0.95	5	0	0	0
ρ	0.16	0.34	0.26	4	0	1	0
R^2	0.95	0.99	0.98	–	–	–	–
				consumption credit			
$\hat{\eta}$	-0.00	0.04	0.01	3	1	0	1
$\hat{\psi}$	0.72	2.84	1.31	4	0	0	1
ρ	0.19	0.79	0.37	5	0	0	0
R^2	0.84	0.99	0.95	–	–	–	–
				corporate credit			
$\hat{\eta}$	-0.00	0.00	0.00	1	0	1	3
$\hat{\psi}$	0.22	1.17	0.80	5	0	0	0
ρ	0.16	0.41	0.26	5	0	0	0
R^2	0.61	0.99	0.89	–	–	–	–

Notes: Estimation period: August 2003 – August 2009 ($NOB = 73$). † — unweighted mean of the estimated coefficients. * — number of estimated coefficients falling in the specified range of statistical significance.

While it is common in the literature to focus on either non-performing loans or loan loss provisions as a measure of distress, the model here combines the two into one system. The equation for loan loss provisions, consistent with equation 1, regresses potential losses on the outstanding stock of non-performing loans proportionally to the loss given default rate:

$$L_t^{b,p} = \lambda^{b,p} e^{-\kappa^{b,p}\tilde{z}_{t-2}} N_t^{b,p} - M_{t-1}^{b,p} + \phi^{b,p} + \nu_t^{b,p}. \tag{5}$$

Equation 5 is a simplification of the actual provisioning process. It tries to find the average level of provisioned losses rather than the exact number period by period because provisioning decisions are discretionary and cannot be explained with a simple model without knowing a bank's policy on making provisions. The expression of the loss given default, $\lambda e^{-\kappa\tilde{z}_t}$, is time variant, and dependent on the asset prices, proxied by the normalised real estate price index, $\tilde{z}$. This is obtained by normalising the ratio of real estate and consumption prices, Z and P^C respectively: $\tilde{z}_t = \left(Z_t/P_t^C - (\overline{Z}/\overline{P^C}) \right) / \sigma(Z/P^C)$. λ is a constant parameter to be estimated and it represents LGD when real estate prices equal their average, $\tilde{z} = 0$. [14] The inclusion of the asset prices enables the rise in the LGD caused by the decrease in the value of collateral and vice versa to be mimicked. Parameter κ shows the importance of real estate prices in every equation.

Variable M stands for written-off loans as in equation 1. Having M in both equations keeps them consistent in that after a loan has been written off, it no longer appears either in the pool of non-performing loans nor in loan loss provisions. Nevertheless, M is excluded from the estimated equations because its variation is close to zero, as stressed before. Parameter ϕ is the bank specific intercept. It mostly stands for the periods when banks face a very low level of non-performing loans but still maintain buffers for loan losses that might occur.

Superscript p is equivalent to m, which was used to identify sectors before. The only difference is that in addition to the sectors already defined, p also includes general provisions, $L^{b,g}$, that is $p \in \{c, f, g, h\}$. General provisions are drawn from the overall stock of non-performing loans, defined as $N_t^{b,g} = \sum_{m \in \{c,h,f\}} N_t^{b,m}$. The estimation results for the loan loss provisions equations are presented in Table 3.

[14] Alternatively λ could also be derived from the loan-to-value ratio and the age of the loan instead of econometric estimating, as is done by Coleman et al. (2005). This method cannot be used here, because it requires micro data from a credit register, which does not exist for Estonian banks.

Table 3: Summary of estimation results: Loan loss provisions

	Min.	Max.	Mean[†]	p^* $[0, 0.01)$	$[0.01, 0.05)$	$[0.05, 0.1)$	$[0.1, \infty)$
				mortgage			
$\hat{\lambda}$	0.03	0.17	0.09	4	0	0	0
$\hat{\kappa}$	0.03	0.47	0.35	4	0	0	0
$\hat{\phi} \times 10^{-2}$	0.00	0.59	0.30	2	0	0	0
R^2	0.93	0.99	0.97	–	–	–	–
				consumption credit			
$\hat{\lambda}$	0.02	0.36	0.21	4	0	0	0
$\hat{\kappa}$	0.00	0.41	0.21	4	0	0	1
$\hat{\phi} \times 10^{-2}$	0.00	0.70	0.35	2	0	0	0
R^2	0.84	0.99	0.93	–	–	–	–
				corporate credit			
$\hat{\lambda}$	0.03	0.40	0.17	5	0	0	0
$\hat{\kappa}$	0.01	1.22	0.52	3	1	0	1
$\hat{\phi} \times 10^{-2}$	0.20	3.84	2.02	2	0	0	0
R^2	0.94	0.99	0.96	–	–	–	–
				general provisions			
$\hat{\lambda}$	0.04	0.48	0.20	3	0	0	0
$\hat{\kappa}$	0.43	1.10	0.76	3	0	0	0
$\hat{\phi} \times 10^{-2}$	0.12	1.15	0.64	2	0	0	0
R^2	0.93	0.99	0.97	–	–	–	–

Notes: Estimation period: August 2003 – August 2009 ($NOB = 73$). † — unweighted mean of the estimated coefficients. * — number of estimated coefficients falling in the specified range of statistical significance.

Estimates of κ indicate the relevance of real estate prices in determining the loss ratio for the banks, as it appears statistically significant in most equations. Estimated λ values show that LGD has been 0.16 on average across banks and sectors (unweighted statistics). Given that general provisions are also drawn from the same pool of nonperforming loans as specific provisions, the effective loss ratio for a bank is higher than the average of the estimated λ values. The effective LGD for the bank b is given by $\sum_m [(\lambda^{b,m} + \lambda^{b,g}) N_t^{b,m}]/(N_t^b)^{-1}$, where $N_t^b = \sum_m N_t^{b,m}$. When real estate prices were at their peak in 2007, effective LGD varied between 0.12 ... 0.3 across the banks, then the fall of real estate prices pushed the LGD range up to 0.3 ... 0.7.

4. Uses of the model

The complete credit risk model consists of 142 equations and identities, all of which are listed in Appendix 1. In addition to equations 2 and 5, the model also includes identities for adding together non-performing loans and loan loss provisions across banks and sectors. In calculating the NPL and LLP ratios out of the sample it is assumed that the growth of each bank's exposure in each of the three sectors equals the average predicted credit growth in the same sector. Credit growth projections are exogenous to the system and they originate from Eesti Pank's forecast model. However, these paths are consistent with the overall macro-economic development because the unemployment rate, GDP growth and inflation are also drawn from the financial development within the forecast model.

Sensitivity analysis

The credit risk model allows investigation into how the related macro variables change credit clients' payment performance and the health of banks' portfolios. Each macro indicator's impact can be explored in isolation to assess its pure effect or it can be looked at in combination with others. Table 4 summarises how the PD, NPL and LLP ratios deviate from their baselines in response to changes in the macro environment. All the macro indicators are shocked permanently by increasing/decreasing their values from their baselines by one standard deviation and by one percentage point/100 base points (signs of shocks are defined so that they would increase the measures of distress). Due to the nonlinearity of the PD function reactions to shocks depend on the initial baseline values of macro variables — shocks hit the financial system harder the further off the macro variables are from their averages. In this exercise all macro variables are initially set to equal their historical means. [15]

The advantage of defining shocks in standard deviations is that it gives better comparability for the resulting reactions. The alternative, defining shock size in percentage points, makes it intuitively easier to understand the magnitude of reactions. All deviations are presented in percentage points and are measured 1.5 years after the occurrence of a shock. This is a sufficient time horizon to let all of the impact to pass through, as macro variables do not affect

[15] The (IMF, 2002) suggests setting a shock size of two standard deviations, which is more appropriate for capturing most of the risk — the probability of a two standard deviation shock is approximately two percent if a normal distribution is assumed. Some studies propose even larger shocks in order to simulate structural breaks in economic relationships and shocks that might never have occurred before but are plausible (Chuhan, 2005). Here one standard deviation serves the goal of comparability of responses.

PD rates instantly but with a considerable time lag; furthermore, interest rates on the stock of loans adjust to changes in the policy rate sluggishly (as will be discussed later).

The responses of the average PD, NPL and LLP ratios to the historical variances of macro indicators highlight the importance of economic growth as the main factor determining the ability of banks' credit clients to service their obligations, *ceteris paribus*. Unemployment clearly remains less important, on average. Shocking both of them by $1pp$ gives a smaller difference in responses, although a shock to output growth still remains twice as damaging even if its higher volatility is disregarded. The reason for this is straightforward — economic activity represents the stream of income for both households and companies, whereas changes in the employment rate only hit households severely.

Changes in the inflation rate have no great importance either. This is in line with the estimation results of equation 3, which showed that short term fluctuations in inflation are not significant in the calculation of the real interest rate but the long term average inflation rate is.

Table 4: Reactions to Changes in Macro Indicators (deviation in pp's.)

variable (shock size)	PD	NPL ratio	LLP ratio
unemployment rate ($+\sigma(u)$ / $+1pp$)	0.05 / 0.02	0.16 / 0.06	0.07 / 0.03
economic growth ($-\sigma(y)$ / $-1pp$)	0.30 / 0.03	1.08 / 0.11	0.55 / 0.06
inflation ($-\sigma(\pi)$ / $-1pp$)	0.04 / 0.01	0.15 / 0.04	0.01 / 0.00
Euribor ($+\sigma(i^*)$ / $+100bp$)	0.17 / 0.14	0.53 / 0.46	0.24 / 0.21
combined effect$^{(a)}$	0.74 / 0.22	2.52 / 0.72	1.08 / 0.31
combined effect$^{(b)}$	0.74 / 0.22	2.52 / 0.72	2.45 / 0.33

Notes: Standard deviations $\sigma(u) = 2.65$, $\sigma(y) = 7.71$, $\sigma(\pi) = 3.74$, $\sigma(i^*) = 1.14$, $\sigma(Z) = 5.27$ (based on common sample: January 2002 – June 2009). $^{(a)}$ with no change in real estate prices, $^{(b)}$ with real estate prices falling by one standard deviation/one percent.

Interest rate changes are mimicked by shocking the 6-month Euribor, which is the base interest rate of the euro-denominated loans and therefore relevant from the policy perspective. [16] The pricing of new loans is done by setting a risk premium, ϑ, on top of the Euribor rate, i^*: $i_t^m = i_t^* + \vartheta_t^m$. Given this pricing mechanism, Euribor changes are reflected in i instantly.

Most of the existing contracts are repriced twice a year, meaning a complete pass through of Euribor into the interest rate on stock takes about six months.

[16]About 87% of all loans are euro denominated. The highest share of euro-based loans is in the mortgage sector at 95%, while in corporate and consumption credit the shares are 88% and 36% respectively.

Equation 6 models the movement of the interest rates on stock, i^s, which in principle is the weighted average of repricing the existing loans and the interest rate of new loans, i:

$$i_t^{s,m} = \mu^m + \chi_t^m[i_{t-1}^{s,m} + \gamma(i_t^* - i_{t-6}^*)] + (1 - \chi_t^m)i_t^m. \tag{6}$$

Parameter γ in equation 6 reflects the proportion of contracts revised each month. If all the contracts were in euros and repriced after every six months, γ would be 1/6. A parameter value less than that shows that not all the contracts are related to the euro. γ can also be less than 1/6 due to contracts with interest rates fixed for a longer period. These are the reasons why the estimation of equation 6 shows that γ is 0.12/0.13 (see Table 5).

Table 5: Estimation results: Interest rates on stock

	Mortgage	Consumption	Firms
$\hat{\mu}$	-0.00	-0.00^{***}	-0.00^{***}
$\hat{\gamma}$	0.13^{***}	0.13^{***}	0.12^{***}
$\hat{\theta} \times 10^{-1}$	-0.35^{***}	-0.25^{***}	-0.26^{***}
$\hat{\alpha} \times 10^{-2}$	0.34^{***}	0.13^{***}	0.12
R^2	0.99	0.91	0.99

Notes: Estimation period: April 2003 – June 2009 ($NOB = 75$). ***, ** and * denote statistical significance at levels of 1%, 5% and 10% respectively.

Weighting parameter, χ_t^m, is time variant and depends on credit growth, $\Delta ln(E_t^m)$: $\chi_t^m = [1 + e^{\theta^m + \alpha^m \Delta ln(E_t^m)}]^{-1}$. At times of faster credit growth the share of the interest rate from new loans increases and it falls in times of slower credit growth. Logistic form keeps the share between zero and one, in the sample χ ranges between 0.8 and 0.95 in all sectors.

A rise in the Euribor rate has considerable effect on measures of distress. In the current set of the macro variables interest rates stand out as the second most significant factor, scaling up to about 2/3 of the effect of output growth (based on their variation). Furthermore, if the last two recession years were not included in the sample for calculation of variation, $\sigma(y)$ would be only 1/5th of the currently used value and interest rates would stand out as the main driver of PD.

The ongoing economic downturn is quite exceptional and severe, resulting from a number of unfavourable developments occuring at the same time: local credit boom and bust, falling real estate prices, the global financial crisis, and low foreign demand. These have led to a significant fall in the growth rate, which dropped to almost -16% in the second quarter of 2009. The short sample gives a lot of weight to the period of economic recession and therefore probably overestimates the variation of output growth. Given this, interest rates should be considered as at least as important source of credit risk as general economic activity. If cyclical swings become smaller over the course of economic development and maturation, a sudden increase in credit costliness would be more harmful than a sudden fall in economic activity, in contrast to what was experienced during the last crisis.

One implication emerging from the shock simulation exercise is that responses to each single shock do not add up together to the combined effect, caused when all the shocks hit the economy at the same time. The non-linear PD equation therefore also implies that each determinant is potentially more vulnerable to changes the further off the *other* macro indicators are from their respective means. This sort of interdependence has clear economic logic — for example, unfavourable or contradictory interest rate shocks are regarded as more damaging when the economic growth rate is lower, and the opposite also holds true.

Deterministic scenario analysis

Although it illuminates the individual macroeconomic determinants, sensitivity analysis only gives a partial view of how credit clients are affected by changes in the economic environment. In the real world all the variables are interlinked. Shocking only one factor at a time disregards the endogenous evolution of the others, which may result in the reaction being underestimated. The same could be true because of the non-linearities built into the system, as shown in the section above. Consistent paths or scenarios for macro indicators can be produced in several ways, typically by using a macroeconomic model, building a stand-alone VAR model of the required macro indicators, or linking a VAR of macro variables directly to the stress test model.

The advantage of using structural macroeconomic models to create scenarios is that they are already built for forecasting and designed to show how shocks materialise (Quagliariello, 2009). This is one of the key characteristics that the model has to comply with, in addition to many others, as discussed by Bårdsen et al. (2006). Their list of relevant model characteristics also includes aspects such as contagion, default, missing financial markets, heterogeneous agents, macroeconomic conditions, structural micro-foundations, empirical tractability, suitability for testing and the inclusion of money, banks, liquidity and default risk. However, after comparing several classes of models (RBC, DSGE, DAE and SVAR) Bårdsen et al. (2006) conclude that no single model can be expected to capture all the risk factors. Building a model with all the features would be extremely complicated, probably impossible and highly inefficient. A suite of models could be used instead.

It is widely recognised that creating a scenario is the most difficult and controversial aspect of stress testing (see for example Blaschke et al. (2001)). The following example uses Eesti Pank's official forecast scenarios from Autumn 2009 to define hypothetical paths for economic development (see Eesti Pank (2009) for more details). These scenarios are not specially designed to test the banking sector under stress in the conventional meaning of stress testing, but nevertheless these are scenarios of severe recession and could be interpreted as the real economy's responses to the most adverse shocks that the Estonian banking sector has ever had to cope with. The strength of using the official forecasts is that they give a fully consistent and thoroughly elaborated view of the future outcomes, which in this particular case involve a projection of a substantial fall in GDP, increased unemployment, deflationary pressures and so forth. As a result of this, the essence of the forecast scenarios is fairly similar to the traditional stress testing exercise, although it does not explicitly explore the limits of the banks.

The forecast scenarios under consideration are produced with the upgraded version of Eesti Pank's macro-econometric forecast model EMMA (Kattai, 2005), which is subject to expert adjustments in the official forecast procedure. The model does not cover all the necessary characteristics outlined by Bårdsen et al. (2006) but still meets some important criteria. Firstly, it is used for forecasting and policy analysis on a regular basis, meaning it has proved its robustness in practice. Secondly, the model also incorporates such financial variables as consumption credit, mortgage loans and corporate credit, and the interest rates for these loans, and so the predicted macro variables are already conditioned on financial developments and the scenarios generated are consistent in this respect. Thirdly, default probabilities are not included in the forecast model but they are directly linked in the credit risk model. There are also some aspects that are not relevant in Estonia, for example due to the high ratio of foreign ownership, the interbank market in Estonia is so thin that there is no need to have heterogeneous agents. [17] In connection with this contagion is also not of high importance (at the local level).

Figure 2 illustrates how the average PD, NPL and LLP rates perform under the *Base* and *Negative Risk* forecast scenarios. The average PD rate falls in both scenarios, by less when the *Negative Risk* materialises, as may be expected. However, PD does not fall to the pre-crisis level even by the end of the forecast period and even if the more positive scenario is considered. The factor keeping the average PD rate higher (*ceteris paribus*) is the increased indebtedness in the economy.

[17] Bårdsen et al. (2006) justify the need for heterogeneous agents with the argument that if all the banks were identical, there would be no incentive for them to trade with each other.

Non-performing loans and loan loss provisions behave in a similar manner — mounting PD keeps them at high rates until they peak around the end of 2009, after which the tensions are relieved. Given that the average PD does not fall to its pre-crisis level, the average NPL and LLP ratios also do not return to the levels they were at before 2008. Sources of improvement lie in the recovery of the economy. Although the unemployment rate remains above its historical average, a rising economic growth rate and increased earnings are sufficient to reverse climbing PD rates, while at the same time predicted low interest rates reduce loan servicing costs and therefore lower risks.

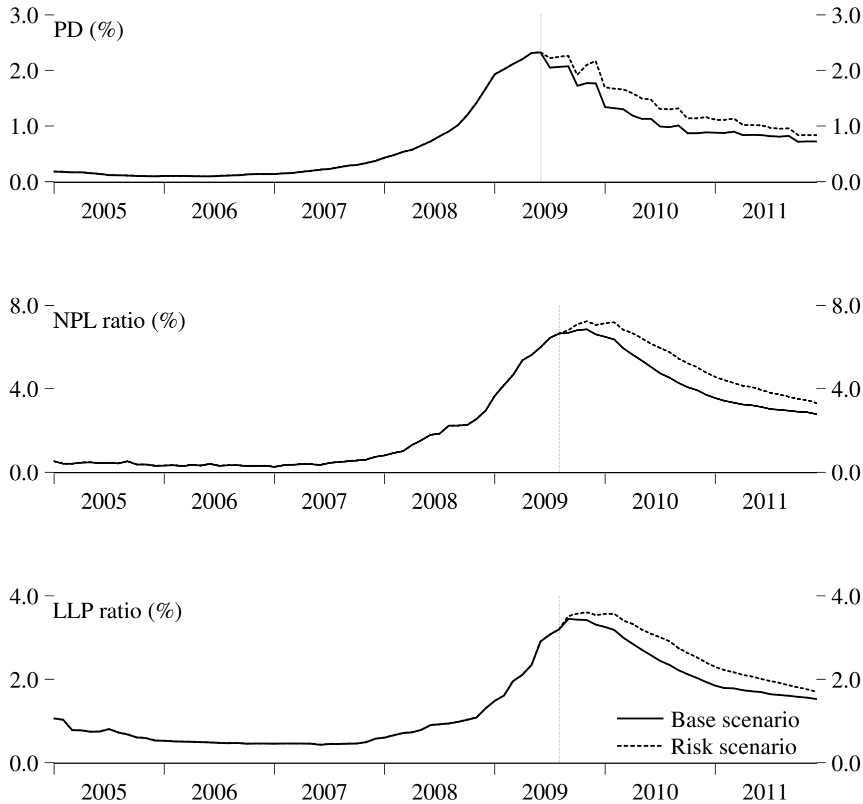


Figure 2: PD, NPL and LLP ratios in the context of the *Base* and *Negative Risk* scenarios.

It is worth noting that according to the model simulation, stabilisation is considerably slower than the worsening of the financial indicators at the emergence of the crisis. This is perfectly in line with the argument that it takes clients longer to recover from the initial shock because they may have run down their savings/resources before they became insolvent and their contracts started to show up as non-performing.

Stochastic scenario analysis

Structural macroeconometric models are useful tools for providing easily interpretable scenarios, but they are barely applicable for analysing uncertainty and producing probabilistic paths for banks' balance sheet items. This applies when the credit risk model contains nonlinearities, as like in this paper. In the presence of nonlinear functions, probabilistic distributions are traditionally obtained by running stochastic (Monte Carlo) simulations (see for example Blaschke et al. (2001)). Macroeconomic models are computationally too demanding to run numerous simulations, thus computationally less demanding VAR models are typically used.

In the present paper all the macro variables that would otherwise have been simulated with the macroeconomic forecast model — GDP growth, inflation, unemployment and the interest rates of the sectors — are compounded into a VAR(2) model:

$$\boldsymbol{X}_t = \boldsymbol{B}_0 + \boldsymbol{B}_1\boldsymbol{X}_{t-1} + \boldsymbol{B}_2\boldsymbol{X}_{t-2} + \boldsymbol{\varepsilon}_t, \tag{7}$$

where $\boldsymbol{X}$ is a vector of macro variables, $\boldsymbol{B}$ is a matrix of the related coefficients and $\boldsymbol{\varepsilon}$ is a vector of error terms (see the more detailed view of the VAR model specification in Appendix 2 and a table with estimation results in Appendix 3). Equations 2 and 5 produce error vectors $\boldsymbol{\omega}$ and $\boldsymbol{\nu}$:

$$\boldsymbol{\Upsilon}_t = \boldsymbol{\eta} + \boldsymbol{\psi}\left((\boldsymbol{I} \otimes \boldsymbol{i})\boldsymbol{\Pi}_{t-2}\right) + (\boldsymbol{\Theta}\boldsymbol{\Gamma}_{t-2})\boldsymbol{\Upsilon}_{t-1} + \boldsymbol{\omega}_t, \tag{8}$$

$$\boldsymbol{L}_t = (\boldsymbol{\lambda}\boldsymbol{Z}_{t-2})\boldsymbol{N}_t + \boldsymbol{\phi} + \boldsymbol{\xi}T_t + \boldsymbol{\nu}_t, \tag{9}$$

which in the combination with $\boldsymbol{\varepsilon}$ gives (6+15+20)$\times$1 vector of errors, $\boldsymbol{Q}$:

$$\boldsymbol{Q} = \begin{pmatrix} \boldsymbol{\omega} \\ \boldsymbol{\nu} \\ \boldsymbol{\varepsilon} \end{pmatrix} \sim N(0, \boldsymbol{\Omega})$$

with 41$\times$41 variance-covariance matrix $\boldsymbol{\Omega}$ (see the more detailed presentation of equations 7, 8 and 9 in Appendix 4):

$$\boldsymbol{\Omega} = \begin{pmatrix} \boldsymbol{\Omega}_{\omega} & \boldsymbol{\Omega}_{\omega,\nu} & \boldsymbol{\Omega}_{\omega,\varepsilon} \\ \boldsymbol{\Omega}_{\nu,\omega} & \boldsymbol{\Omega}_{\nu} & \boldsymbol{\Omega}_{\nu,\varepsilon} \\ \boldsymbol{\Omega}_{\varepsilon,\omega} & \boldsymbol{\Omega}_{\varepsilon,\nu} & \boldsymbol{\Omega}_{\varepsilon} \end{pmatrix}.$$

Simulations are executed by picking random draws of $U_{t+a} \sim N(0,1)$ into a 41×1 vector $\boldsymbol{U}_{t+a}$, which is then transformed into a vector of innovations of the macro indicators, the NPL and LLP rates. This transformation is achieved by scaling the draws to match the variance-covariance structure of matrix $\boldsymbol{\Omega}$ by multiplying the vector of random numbers by the Cholesky factor $\boldsymbol{A}$, where $\boldsymbol{\Omega} = \boldsymbol{A}\boldsymbol{A}'$. The result is a vector of correlated innovations, $\boldsymbol{Q}_{t+a} = \boldsymbol{A}'\boldsymbol{U}_{t+a}$, which takes into account correlations between shocks in macroeconomic factors and financial data. Simulated values of $\boldsymbol{Q}_{t+a}$ can then be used to determine the PD, NPLs and LLPs for every t for the specified time horizon ahead. Repeating the procedure in a Monte Carlo routine finally gives simulated probabilistic paths for the model's variables.

A fair criticism of the approach is that the simulation exercise only takes account of the variance-covariance matrix Ω and sets the remaining sources of risk to zero (Kupiec, 1998). The threat of an important risk factor being missed and probable losses underestimated as a result is greater if the risk factor is uncorrelated with the others. No model is ever going to capture all the potential risks, however, but the interpretation of the simulation results must leave some room for some additional, potentially negative shock impacts.

Figure 3 illustrates the outcome of a stochastic simulation exercise. The model has been run 10,000 times up to the end of 2011 with a new set of randomly drawn innovations each time. Average distributions of PD and loan loss provisions rates are plotted as they appear at the ends of the years 2009, 2010 and 2011 (the results do not show the uncertainty of the model's coefficients but only the effect of the innovations). It has been found in the literature that aggregation across different institutions may be a source of an additional error. However the problem is not so pronounced if the harmonised data and the same modelling methodology has been adopted for all institutions (Blaschke et al., 2001), like in this paper [18]

All the distributions share the same pattern, being skewed to the right as is generally common to credit risk, meaning larger losses are less likely to occur. [19] The pattern emerging from the exercise is that the distribution of PD shifts to left in a more distant time horizon. The reason behind this is the recovery of the economy, predicted by the VAR model, where output growth tends to drive the dynamics of the other macro variables too (see Appendix 5 for the impulse responses of the VAR model).

The distributions of the average non-performing loan and loan loss provi-

[18]The additivity problem becomes an issue when stress test results are carried out by individual institutions (banks) and then merged by a central body. It can be avoided by harmonizing methodologies and scenarios.

[19]Right-skewed distribution is caused if the majority of elements in the variance-covariance matrix $\boldsymbol{\Omega}$ are positive.

sion rates also shift to left as a consequence of the lower PD rate. Median values clearly indicate a substantial fall in the expected distress measures in the subsequent years, but it is also worth noting that the right hand side tails of the distribution areas remain stretched out. Skewness increases year by year and despite the leftward shift of the median, the maximum NPL and LLP ratios remain basically unchanged (for example if 1% probability of occurrence is considered). The increased skewness and the widening of the distribution areas mostly reflect higher levels of forecast uncertainty, which increases over time.

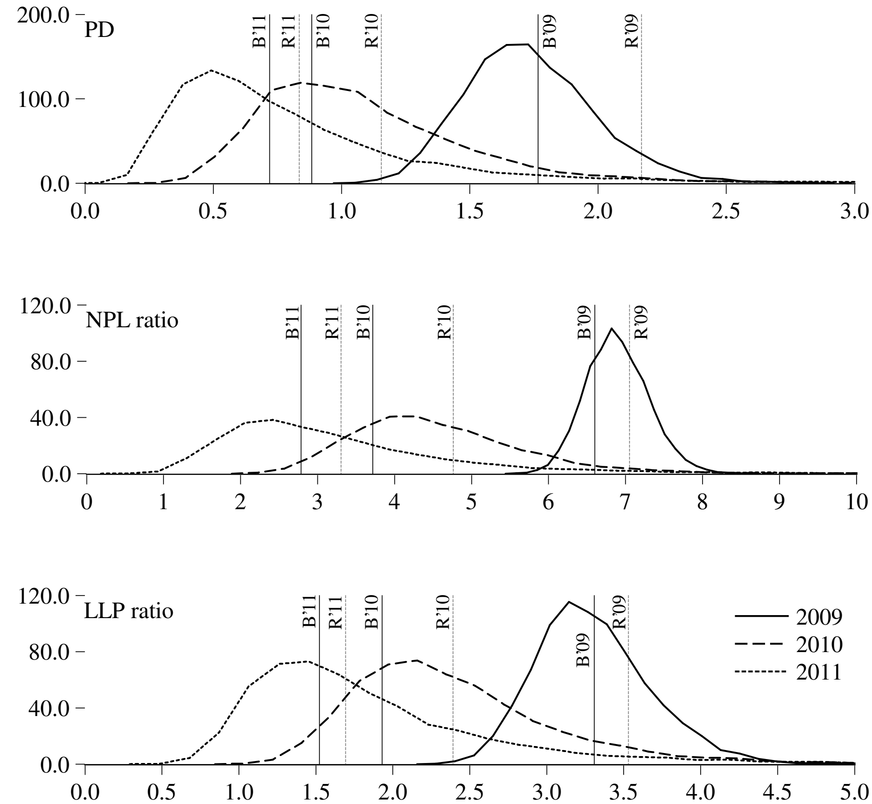


Figure 3: Probabilistic distributions (densities) of the average PD, NPL and LLP ratios at the end of 2009, 2010 and 2011 (%) and comparative outcomes for deterministic scenario analysis ($B'(.)$ and $R'(.)$ denote *Base* and *Negative Risk* scenarios respectively).

Figure 3 also depicts the point estimates of all the plotted variables from the previous exercise, the deterministic scenario analysis. The average PD, NPL and LLP ratios measured at the ends of 2009, 2010 and 2011 (which coincides with the period of probabilistic estimates) depending on whether the

Base or *Negative Risk* scenario is expected to materialise, seem to fit the results of the stochastic simulation pretty well, as all the point estimates lie in their respective distribution areas. The *Base* scenario's point estimates are closer to medians of the distributions, making a better comparison with the VAR-based scenario. This outcome is expected in the sense that the *Base* scenario is subject to relatively fewer expert adjustments compared to the *Negative Risk* scenario.

The good fit of the distribution medians and the *Base* scenario point estimates allows the stochastic simulation results to be used to draw something similar to confidence bands around the official forecast-based outcomes. This has to be done with caution, of course, because the relationship between the two is not straightforward. But the benefit of this sort of interpretation is that it allows to emphasise probabilistic deviations from deterministic paths, which are clearly and meaningfully explained by the narrative of the forecast (unlike the VAR-based forecast, which is not easily explainable).

The macro forecast *Base* scenario predicts that the sudden stop in the output growth in 2009 will be followed by a period of stabilisation in 2010 and by the recovery of growth in 2011 (Eesti Pank, 2009). This is clearly seen in the dynamics of the PD, NPL and LLP ratios in Figure 2, which show reliefs in the distress measures. Risks are reduced not only by the recovery in output growth but also by the lower unemployment rate against the background of relatively low interest rates. But what is left out of the picture is that distributional properties (error margin) change over time. The increases in skewness and the widening of the distribution areas (see Figure 3) imply that under certain unfavourable circumstances loan loss provisions may not fall but may remain the same for the next two years. Although it is highly unlikely in statistical terms, the deterministic scenario analysis is not able to convey numerical estimates of these risks. Uncertainty estimates from the stochastic scenario analysis therefore suggest that in order to be on the safe side, banks' capital buffers should not be significantly decreased even if general macroeconomic condition start to improve. Even if the volume of past due loans becomes less troubling, it may also be that the restructuring of the loans will mean the banks' credit portfolios will be somewhat lower quality and they will remain more vulnerable to future shocks.

5. Conclusions

Eesti Pank's credit risk model draws a link between underlying economic conditions and the health of the banks' credit portfolios. It is able to separate the performance of the four major banks in the market and aggregates the rest of the sector together (the share of the four biggest banks is about 94%). The model also splits each banks portfolio into mortgage, consumption and corporate credit. The health of the credit portfolios in the specified breakdown are characterised by non-performing loans and loan loss provisions, which are direct reflections of the default probabilities of credit clients. The macro indicators that matter for clients' payment performance (default probabilities) are economic growth, the unemployment rate, interest rates, inflation, indebtedness and credit growth.

Sensitivity tests performed with the model indicate that output growth has been a leading factor determining the probability of banks' clients facing difficulties in servicing their debts. This holds both for households and companies, with output acting as a proxy for households' disposable income and for companies' revenues. The effect of interest rate movements has remained less important within the sample. However, the dominance of the income effect is highly exaggerated by the recent downturn in the national economy, which has substantially increased the standard deviation of output growth. After the economy exits the recession, interest rates will probably be at least as important source of risk as output growth; with the one important difference that interest rate movements have a slower impact on distress than do deviations in output growth. Any shock to the base interest rate, Euribor, or to the risk premium takes about a year to transmit fully into default probabilities. The time lag occurs because most loans are repriced only twice a year and clients are able to absorb shocks for some time.

Scenario analysis based on Eesti Pank's most recent forecast (released in Autumn 2009) foresees a relaxation of tensions in the credit market. Default probabilities are expected to fall from early 2010 as a result of macro economic stabilisation and a return to growth. Non-performing loans and loan loss provisions are projected to diminish as a result of this. Stochastic scenario analysis, however, suggests that this improvement should be treated with reasonable caution, because although it becomes less probable over the course of time, it is not inconceivable that provisions will remain at their peak level until the end of the forecast horizon in December 2011 (assuming that they are not written off before then).

The list of possible improvements to the model contains suggestions related to the credit risk model itself and to the performance of stress tests. The first category includes several issues concerning the modeling framework. It was noted in the paper that the constancy of the parameter ρ, the rate at which non-performing loans become performing again, is very strong and most likely in reality the parameter varies over time. Therefore additional effort has to be devoted to exploring possible ways to tie ρ to economic fundamentals or directly to the default rate.

The model could also be improved by the direct inclusion of banks' profits and their capital. Doing so would enable the capital adequacy ratio to be monitored directly, not outside the model as is done at the moment.

One possible improvement concerning the execution of stress tests, or more precisely the macro scenario analysis, would be to introduce feedback from the credit risk model into the macro model. At the moment the macro model is only used to generate consistent macro scenarios but has no second round effects or feedback from the credit risk model. There are linkages in both directions which should not be discarded, in principle, as this fact has also been stressed in the literature on the subject.

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Appendix 1. Equations and identities in the Credit Risk Model

Dependent variable	Equation/Identity	No.
NPL positions	$N_t^{b,m} = (\eta^{b,m} + \psi^{b,m}\Pi_{t-2}^m)E_{t-2}^{b,m}+$	15
	$+\left(1-\rho^{b,m}\right)N_{t-1}^{b,m}$	
Bank's overall NPLs	$N_t^b = \sum_m N_t^{b,m}$	5
NPLs in sector m	$N_t^m = \sum_b N_t^{b,m}$	3
Aggregate NPLs	$N_t = \sum_b \sum_m N_t^{b,m}$	1
LLP positions	$L_t^{b,p} = \lambda^{b,p} e^{-\kappa^{b,p}\tilde{z}_{t-2}} N_t^{b,p} - M_{t-1}^{b,p} + \phi^{b,p}$	20
Bank's overall LLPs	$L_t^b = \sum_p L_t^{b,p}$	5
LLPs in sector p	$L_t^p = \sum_b L_t^{b,p}$	4
Aggregate LLPs	$L_t = \sum_b \sum_p N_t^{b,p}$	1
PD in sector m	$\Pi^m = e^{f(\beta^m,X)}/(1+e^{f(\beta^m,X)})$	3
	$f(\beta^m, X) = -\beta_0^m + \beta_1^m \hat{\tilde{u}}_{t-l} - \beta_2^m \hat{\tilde{y}}_{t-o}+$	
	$+\beta_3^m \tilde{r}_{t-j}^{s,m} + \beta_4^m \hat{\tilde{e}}_t^m - \beta_5^m \hat{\tilde{g}}_t^m$	
Real i-rates	$r_t^{s,m} = i_t^{s,m} - \tau^m \hat{\pi}_t + (1-\tau^m)\overline{\pi}$	
Nominal i-rates, new loans	$i_t^m = \hat{i}_t^* + \hat{\vartheta}_t^m$ or $i_t^m = \hat{i}_t^m$	3
Nominal i-rates on stock	$i_t^{s,m} = \mu^m + \chi_t^m[i_{t-1}^{s,m} + \gamma(\hat{i}_t^* - \hat{i}_{t-6}^*)]+$	3
	$+(1-\chi_t^m)i_t^m$	
	$\chi_t^m = [1 + e^{\theta^m + \alpha^m \Delta ln(E_t^m)}]^{-1}$	
Exposure, bank, sector	$E_t^{b,m} = E_{t-1}^{b,m}\hat{E}_t^m/\hat{E}_{t-1}^m$	15
Banks' portfolio	$E_t^b = \sum_m E_t^{b,m}$	5
Sector's exposure	$E_t^m = \sum_b E_t^{b,m}$	3
Aggregate exposure	$E_t = \sum_b E_t^b$	1
Bank's NPL ratio in sector m	$R_t^{N^{b,m}} = N_t^{b,m}/E_t^{b,m}$	15
Bank's overall NPL ratio	$R_t^{N^b} = N_t^b/E_t^b$	5
NPL ratio, sector m	$R_t^{N^m} = N_t^m/E_t^m$	3
Aggregate NPL ratio	$R_t^N = N_t/E_t$	1
Bank's LL ratio, sector p	$R_t^{L^{b,p}} = L_t^{b,p}/E_t^{b,p}$	20
Bank's overall LL ratio	$R_t^{L^b} = L_t^b/E_t^b$	5
LL ratio in sector p	$R_t^{L^p} = L_t^p/E_t^p$	4
Aggregate LL ratio	$R_t^L = L_t/E_t$	1
Real estate prices	$Z_t = Z_{t-1}$ or $Z_t = \hat{Z}_t$	1

Notes: Variables with a hat ($\hat{\cdot}$) denote series predicted with the macro econometric forecast model or VAR model.

Appendix 2. Matrix representation of the VAR model

$$\underbrace{\begin{pmatrix} y_t \\ \pi_t \\ i_t^h \\ i_t^c \\ i_t^f \\ u_t \end{pmatrix}}_{\boldsymbol{X}_t(6\times 1)} = \underbrace{\begin{pmatrix} b_{1,1}^0 & \dots & b_{1,6}^0 \\ \vdots & \ddots & \dots \\ b_{6,1}^0 & \dots & b_{6,6}^0 \end{pmatrix}}_{\boldsymbol{B}_0(6\times 6)} + \underbrace{\begin{pmatrix} b_{1,1}^1 & \dots & b_{1,6}^1 \\ \vdots & \ddots & \dots \\ b_{6,1}^1 & \dots & b_{6,6}^1 \end{pmatrix}}_{\boldsymbol{B}_1(6\times 6)} \underbrace{\begin{pmatrix} y_{t-1} \\ \pi_{t-1} \\ i_{t-1}^h \\ i_{t-1}^c \\ i_{t-1}^f \\ u_{t-1} \end{pmatrix}}_{\boldsymbol{X}_{t-1}(6\times 1)}$$

$$\underbrace{\begin{pmatrix} b_{1,1}^2 & \dots & b_{1,6}^2 \\ \vdots & \ddots & \dots \\ b_{6,1}^2 & \dots & b_{6,6}^2 \end{pmatrix}}_{\boldsymbol{B}_2(6\times 6)} \underbrace{\begin{pmatrix} y_{t-2} \\ \pi_{t-2} \\ i_{t-2}^h \\ i_{t-2}^c \\ i_{t-2}^f \\ u_{t-2} \end{pmatrix}}_{\boldsymbol{X}_{t-2}(6\times 1)} + \underbrace{\begin{pmatrix} \varepsilon_t^y \\ \varepsilon_t^\pi \\ \varepsilon_t^{i^h} \\ \varepsilon_t^{i^c} \\ \varepsilon_t^{i^f} \\ \varepsilon_t^u \end{pmatrix}}_{\boldsymbol{\varepsilon}_t(6\times 1)}$$

Appendix 3. VAR model estimation results

	y	π	i^h	i^c	i^f	u
$constant$	0.01	6.0	−0.01	0.07	0.00	0.01
	(0.01)	(0.0)	(0.00)	(0.01)	(0.01)	(0.00)
y_{t-1}	1.57	.16	0.06	−0.09	0.06	−0.01
	(0.08)	(0.09)	(0.02)	(0.14)	(0.05)	(0.04)
y_{t-2}	−0.59	−0.11	−0.05	0.01	−0.05	−0.01
	(0.08)	(0.09)	(0.02)	(0.14)	(0.05)	(0.04)
π_{t-1}	−0.08	1.13	−0.01	0.04	0.07	0.06
	(0.08)	(0.09)	(0.02)	(0.15)	(0.06)	(0.05)
π_{t-2}	0.11	−0.19	0.01	−0.03	−0.06	−0.08
	(0.08)	(0.09)	(0.02)	(0.15)	(0.06)	(0.05)
i^h_{t-1}	0.15	0.15	0.98	0.21	0.42	−0.15
	(0.32)	(0.36)	(0.09)	(0.58)	(0.22)	(0.18)
i^h_{t-2}	−0.28	−0.21	−0.06	−0.58	−0.06	0.25
	(0.32)	(0.36)	(0.09)	(0.58)	(0.22)	(0.18)
i^c_{t-1}	−0.15	−0.01	−0.02	0.55	0.04	−0.06
	(0.05)	(0.06)	(0.02)	(0.09)	(0.04)	(0.03)
i^c_{t-2}	0.04	0.06	0.04	−0.09	0.05	0.04
	(0.05)	(0.06)	(0.02)	(0.10)	(0.04)	(0.03)
i^f_{t-1}	−0.23	−0.01	0.01	0.45	0.19	−0.13
	(0.14)	(0.15)	(0.04)	(0.25)	(0.09)	(0.08)
i^f_{t-2}	0.36	0.06	0.09	0.18	0.26	0.07
	(0.13)	(0.14)	(0.04)	(0.23)	(0.08)	(0.07)
u_{t-1}	−0.04	0.22	−0.03	0.24	−0.05	1.38
	(0.15)	(0.17)	(0.04)	(0.27)	(0.10)	(0.08)
u_{t-2}	0.10	−0.19	0.01	−0.40	0.03	−0.46
	(0.15)	(0.17)	(0.04)	(0.27)	(0.10)	(0.08)
R^2	0.99	0.94	0.99	0.70	0.94	0.99

Notes: Estimation sample: March 1999 – June 2009 (124 obs.). Included variables: y — econmic growth, π — inflation rate, i^h — mortgage interest rate, i^c — consumption credit interest rate, i^f — corporate credit interest rate, u — unemployment rate. Standard errors of coefficients are presented in parentheses.

Appendix 4. Matrix representation of non-performing loans and loan loss provisions equations

Non-performing loans

$$
\underbrace{\begin{pmatrix} N_t^{s,h}/E_{t-2}^{s,h} \\ \vdots \\ N_t^{o,f}/E_{t-2}^{o,f} \end{pmatrix}}_{\boldsymbol{\Upsilon}_t(15\times1)} = \underbrace{\begin{pmatrix} \eta^{s,h} \\ \vdots \\ \eta^{o,f} \end{pmatrix}}_{\boldsymbol{\eta}(15\times1)} +
$$

$$
+ \underbrace{\begin{pmatrix} \psi^{s,h} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \psi^{o,f} \end{pmatrix}}_{\boldsymbol{\psi}(15\times15)} \left(\left(\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\boldsymbol{I}(3\times3)} \otimes \underbrace{\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}}_{\boldsymbol{i}(5\times1)} \right) \underbrace{\begin{pmatrix} \Pi_{t-2}^{h} \\ \Pi_{t-2}^{c} \\ \Pi_{t-2}^{f} \end{pmatrix}}_{\boldsymbol{\Pi}_{t-2}(3\times1)} \right) +
$$

$$
+ \left(\underbrace{\begin{pmatrix} 1-\rho^{s,h} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & 1-\rho^{o,f} \end{pmatrix}}_{\boldsymbol{\Theta}(15\times15)} \underbrace{\begin{pmatrix} \frac{E_{t-3}^{s,h}}{E_{t-2}^{s,h}} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \frac{E_{t-3}^{o,f}}{E_{t-2}^{o,f}} \end{pmatrix}}_{\boldsymbol{\Gamma}_{t-2}(15\times15)} \right) \underbrace{\begin{pmatrix} N_{t-1}^{s,h}/E_{t-3}^{s,h} \\ \vdots \\ N_{t-1}^{o,f}/E_{t-3}^{o,f} \end{pmatrix}}_{\boldsymbol{\Upsilon}_{t-1}(15\times1)} +
$$

$$
+ \underbrace{\begin{pmatrix} \omega_t^{s,h} \\ \vdots \\ \omega_t^{o,f} \end{pmatrix}}_{\boldsymbol{\omega}_t(15\times1)}
$$

Order of bank and sector specific variables and coefficients $x^{b,m}$: $x^{s,h}$, $x^{w,h}$, $x^{d,h}$, $x^{n,h}$, $x^{o,h}$, $x^{s,c}$, $x^{w,c}$, $x^{d,c}$, $x^{n,c}$, $x^{o,c}$, $x^{s,f}$, $x^{w,f}$, $x^{d,f}$, $x^{n,f}$, $x^{o,f}$. SEB, Swedbank, Danske, Nordea and the rest of the banks are denoted with s, w, d, n and o. Consumption credit, mortgage loans and corporate credit are marked with c, h and f.

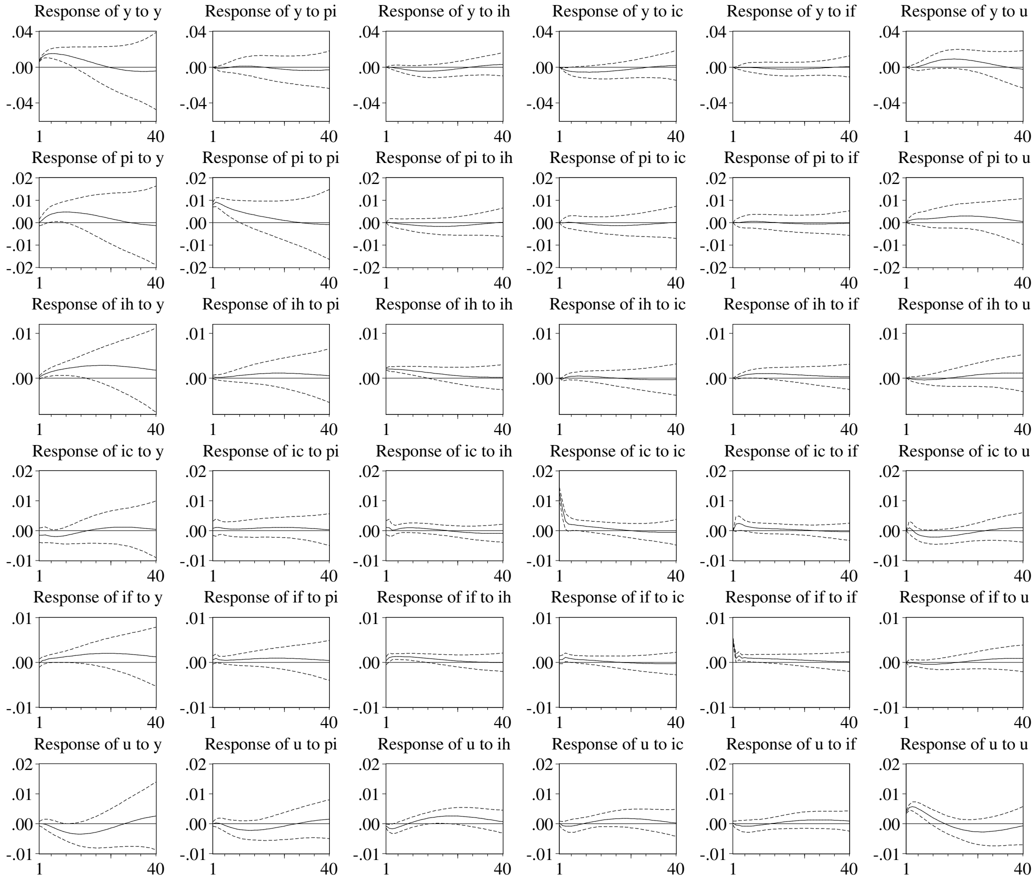
Loan loss provisions

$$\underbrace{\begin{pmatrix} L_t^{s,h} \\ \vdots \\ L_t^{o,g} \end{pmatrix}}_{\boldsymbol{L}_t(20\times 1)} = \left(\underbrace{\begin{pmatrix} \lambda^{b,p} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \lambda^{b,p} \end{pmatrix}}_{\boldsymbol{\lambda}(20\times 20)} \underbrace{\begin{pmatrix} e^{-\kappa^{s,h}\tilde{z}_{t-2}} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & e^{-\kappa^{o,g}\tilde{z}_{t-2}} \end{pmatrix}}_{\boldsymbol{Z}_{t-2}(20\times 20)} \right) \underbrace{\begin{pmatrix} N_t^{s,h} \\ \vdots \\ N_t^{o,g} \end{pmatrix}}_{\boldsymbol{N}_t(20\times 1)} +$$

$$+ \underbrace{\begin{pmatrix} \phi^{s,h} \\ \vdots \\ \phi^{o,g} \end{pmatrix}}_{\boldsymbol{\phi}(20\times 1)} + \underbrace{\begin{pmatrix} \xi^{s,h} \\ \vdots \\ \xi^{o,g} \end{pmatrix}}_{\boldsymbol{\xi}(20\times 1)} T_t + \underbrace{\begin{pmatrix} \nu_t^{s,h} \\ \vdots \\ \nu_t^{o,g} \end{pmatrix}}_{\boldsymbol{\nu}_t(20\times 1)}$$

Order of bank and sector specific variables and coefficients $x^{b,m}$: $x^{s,h}$, $x^{w,h}$, $x^{d,h}$, $x^{n,h}$, $x^{o,h}$, $x^{s,c}$, $x^{w,c}$, $x^{d,c}$, $x^{n,c}$, $x^{o,c}$, $x^{s,f}$, $x^{w,f}$, $x^{d,f}$, $x^{n,f}$, $x^{o,f}$, $x^{s,g}$, $x^{w,g}$, $x^{d,g}$, $x^{n,g}$, $x^{o,g}$. SEB, Swedbank, Danske, Nordea and the rest of the banks are denoted with s, w, d, n and o. Provisioned consumption credit, mortgage loans, corporate credit and general provisions are marked with c, h, f and g.

Appendix 5. VAR model impulse responses



Notes: Responses to Cholesky 1 S.D. innovations with ±2 S.E. bounds. Order of Cholesky decomposition: economic growth, y; inflation rate, π; mortgage interest rate, i^h; consumption credit interest rate, i^c; corporate credit interest rate, i^f and unemployment rate, u. Standard errors are based on 10000 Monte Carlo simulation repetitions.

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