John-Tagore Tevet

CONSTRUCTIVE PRESENTATION OF THE GRAPHS:

A SELECTION OF EXAMPLES

CONTENTS

Non profit research group S.E.R.R., Tallinn, Estonia
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INTRODUCTION

<u>Abstract:</u> ***Constructive presentation*** of the graphs designed for recognition the *structure, its symmetry properties (orbits), attributes, changes, successions and systems* of the graphs, be founded on special concepts and be realized by corresponding algorithms. It constitutes a ***structure semiotic*** approach to the graphs. ***Structure*** of a graph mean there a *complete invariant* of isomorphic graphs and ***semiotics*** be expressed as a *sign system* of local invariants, that present the structure in a ***constructive form***. There are presented 34 examples with corresponding propositions and comments.

To date the graph theory, in spite of variety of the problems and approaches, has been dominated by a certain "Königsberg attitude" with its emphasis on walks, paths, cycles, directed, Eulerian- and Hamiltonian graphs and in the flows in theirs. Unfortunately remain the aspects of structural, systemic and symmetry properties to the background.

We take the interests for:

(i) What is the structure?
(ii) What is the symmetry of structure?
(iii) What are the attributes of structure?
(iv) What are the changes of structure?
(v) What is a system of structures?
(vi) What is the semiotics of structure?

Graphs are constructive objects that can be treated on various aspects. Layers of conceptualization have enlarged our understanding to their complexity. However, attempts to find the "truth" about graphs remain contentious. Presented application of semiotic testing may be an innovative response to what J. Mayer [1976] noted as the introverted condition of graph theory and give it new intellectual state. It is a usual discovery of the graphs.

Structure semiotics no be interested in *directed- and multigraphs, walks, traversability, flows, planarity, colorability, coverings, specters etc.* At the same time by the structural approach of the graphs arise any specific research objects.

The foundations of constructive presentation are explained by a system of specific *definitions and conceptions* (see 'Structure Semiotic Approach to the Graphs' pp 2-9 by (PDF) http://ester.nlib.ee/ or www.graphs.ee pp 1-8. The conceptions are developed to corresponding *algorithms* for opening in a simple way the structure and its attributes (see pp 24-30 or 36-40 correspondingly).

The selection of *examples* with *comments* and *propositions* express there the processing results of algorithms, where give much attention to *symmetry properties,* particularly to bisymmetry of structure. As a rule, we *recognize* the structural attributes of the graphs, such as:

1) ***Structure of a graph*** and its ***complement*** in *constructive form*.
2) ***Orbits*** of ***vertices*** and ***vertex pairs*** of structure and its complement.
3) ***Symmetry signs*** and ***kinds of structure***, such as *complete-, bi-, mono-, polyi-, local- and 0-symmetry*.
4) ***Valences*** and ***valence regularity*** (i.e. custom regularity).
5) ***Distances*** and ***distance regularity***, where all the vertices have on the equal distance ***–d*** an equal number ***m*** vertices.
6) ***Girths*** and ***girth regularity***, where all the vertices belong to some girth with equal perimeter ***(+d+1)*** an equal number ***p***≥1 times.
7) ***Cliques*** and ***clique regularity***, where all the vertices belong to a clique with equal power ***n***; we take structure *n-clique-regular* also then, if the some vertices belong to *(n+a)*-clique, because *n*-clique is its sub-clique.
8) ***Strong regularity***, that mean a state (*k*,***a***,***b***), where *each adjacent pair of k-valence-regular structure has* ***a***≥0 *common adjacent vertices* and *each disadjacent pair* ***b***≥1 *common adjacent vertices*.
9) ***Partitions***, such as *bipartite, tripartite etc* and the ***lists of parts***.
10) ***Orbit structures*** as real parts of a structure and its complement, i.e. partial structures whose edges correspond to *pair signs of a fixed pair orbit* of the structure.
11) ***Adjacent structures,*** i.e. *greatest substructures* and *smallest superstructures* of the structure, where their number cancel to the number of pair orbits ***N.***
12) ***Structural measures,*** i.e. values by diversity of structural attributes.

The examples are selected so, that all the essential structural properties of symmetric and non-symmetric graphs are presented.

ATTRIBUTES OF CONSTRUCTIVE PRESENTATION

On the structural aspect is a ***graph*** only a list of adjacent vertices or adjacent matrix. The „figures" or „diagrams" of graphs are there presented in minimum. The aim is to present the essence of graph structure. For opening structural attributes is suitable to treat the *structure with its complement together*. For structural research elaborated two algorithm-complete: *A) Structure algorithm* and *B) System algorithm.* There be limited with explaining the input (initial data) and output (processing results) of structure algorithm.

INPUT: List of adjacent vertices ***L*** of a graph**.**

STRUCTURE ALGORITHM: 1) Forming the list of adjacent vertices of complement. 2) Identification the local invariants, i.e. *pair signs* of *pair graphs* and their lexicographical ordering to the form of *sign matrix* ***W***. 3) Recognition the essential attributes. 4) Forming the lists of adjacent vertices of pair graphs.

OUTPUT: Sign matrix ***W***, that open the structure with exactness up to isomorphism with corresponding attributes together and recognize all the structural attributes in canonical.

Introductive example in standard form with shorts explanations.

Graph-structure ***G282*** *(7.6.24)* and its complement ***G1203*** *(7.15.24).* The ordering number of graph corresponds to its number in Graph Atlas, the number in brackets to its number in the system of structures with 7-vertices.

INPUT: List of adjacent vertices ***L*** of graph ***G282***:

```
1 – 2, 3, 7;
2 – 1, 3;
3 – 1, 2;
4 – 5, 6;
5 – 4;
6 – 4;
7 – 1;
```

OUTPUT 1: Pair signs and sign matrix ***W*** with *u*- and *s*-signs of graph ***G282*** and its complement:

A:-2.3.2; B:-0.2.0; ***C:+1.2.1; D:+2.3.3.***

1	**2**	**3**	**3**	**4**	**4**	**5**	***i***		***k***	
1	4	2	3	5	6	7		*AB**CD***		12345
0	*-B*	***D***	***D***	*-B*	*–B*	**C**	1	*03**12***	**1**	00201
	0	*-B*	*–B*	**C**	**C**	*-B*	4	*04**20***	**2**	00020
		0	***D***	*-B*	*–B*	*-A*	2	*13**02***	**3**	10100
			0	*-B*	*–B*	*-A*	3	*13**02***	**3**	10100
				0	*–A*	*-B*	5	*14**10***	**4**	01000
					0	*-B*	6	*14**10***	**4**	01000
						0	7	*23**10***	**5**	10000

A:-2.6.12; B:-2.6.10; C:-2.5.7;
D:+2.3.3; E:+2.4.5; F:+2.4.6; G:+2.5.8; H:+2.5.9; I:+2.6.11; J:+3.7.15.

1	**2**	**3**	**3**	**4**	**5**	**5**	***i***		***k***	
7	1	5	6	4	2	3		*ABC**DEFGHIJ***		12345
0	*-C*	***H***	***H***	***E***	***G***	***G***	7	*001**0102200***	**1**	00212
	0	***D***	***D***	***J***	*-C*	*-C*	1	*003**2000001***	**2**	00210
		0	***I***	*-B*	***F***	***F***	5	*010**1020110***	**3**	11102
			0	*-B*	***F***	***F***	6	*010**1020110***	**3**	11102
				0	***D***	***D***	4	*020**2100001***	**4**	11002
					0	*–A*	2	*101**1021000***	**5**	10210
						0	3	*101**1021000***	**5**	10210

Pair signs: Pair sign **±*d.n.q*** open: –***d*** – *distance* between vertices or the length of path, ***n*** – number of vertices in pair graph, ***q*** – number of edges in pair graph. Pair sign *–0.2.0* is *disconnecting sign* and ***+1.2.1*** a *path link sign*. **+*d***>1 is *girth sign*, that open the *collateral distance* between adjacent vertices, where ***d***+1 is the *length of girth*, for example, **+*d=2*** is *3-girth-* or *triangularity sign*. Sign ***+2.3.3*** on *3-clique-*, ***+2.4.6*** *4-clique sign* (if after to **+2**[le] the ***n*** and ***q*** correspond to conditions of *n*-clique), ***J:+3.7.15*** is *4-girth sign*, where the ***n****=7* vertices and ***q****=15* adjacent vertices of pair graph $g \subset G$ belong to *4-girths*. If n equal to the number of vertices and q to the number of edges in graph G, as now, then it is *complete pair sign*.

u-signs: Sign $u_i = u_{i1},...,u_{ip},...,u_{iP}$, whose elements u_{ip} are by pair signs, $d.n.q_1 < ... < d.n.q_p < ... < d.n.q_P$, lexicographically ordered presents their corresponding number in the row W_i of sign matrix, is ***u-sign*** of vertex v_{ij}. *u*-signs are presented in the column *AB**CD…*** of matrix W.

s-signs: Sign $s_i = s_{i1},...,s_{ik},...,s_{iK}$, whose elements s_{ik} present the number of adjacent vertices (of pair(+)signs) in the class W_k of vertex v_i is ***s-sign*** of vertex v_i. *S*-signs are presented in the column 12345 of matrix W.

Vertex orbits: Vertex v_i orbits ΩV_k are presented in column k of matrix W.

Pair orbits: Pair $v_i v_j$ orbits ΩR_n are presented in the intersection $W_{ki,kj} = W_{ki} \cap W_{kj}$ of decomposed sign matrix W. For example, in the partial matrix $W_{1.3}$ is by sign D is opened a two-element orbit of adjacent pairs *1-2 and 1-3*, and in the partial matrix $W_{3.4}$ by sign *–B* a four-element orbit of disadjacent pairs.

OUTPUT 2: Common invariants and measures of structure and its complement:

Symmetry	|V|	|R|	K	N	SVV	SV	SRV	HR	SR	aut	PS
Local-symmetry	**7**	**21**	**5**	**12**	$1^3 2^2$	*0.204*	$1^5 2^6 4^1$	*1.035*	***0.217***	4	*156/7752*

$|V|$ – number of vertices; $|R|$ – number of vertex pairs; K – number of vertex orbits; N – number of pair orbits.

Symmetry signs: *SVV* – ***sign of vertex symmetry***, where large numbers **N** present the powers of vertex orbits and small upper numbers ^{N} the number of vertex orbits; ***SRV – sign of pair symmetry***, where large numbers **N** present the power of pair orbits and small upper numbers ^{N} the number of pair orbits.

Measures: ***SV*** – ***value of vertex symmetry***, $0 \le SV \le 1$; ***HR*** – value of inner diversity or ***information capacity***; ***SR*** – ***value of pair symmetry***, $0 \le SR \le 1$; ***aut*** – number of ***automorphisms***; ***PS*** – ***existence probability***.

OUTPUT 3: Distinguishing invariants and measures of structure and its complement:

G	|E|	k	N^+	N^-	P	CL	MC	DM	SEV	SE	DEG	CPX	TRA	BRA	pro
282	6	2	4	8	4	3	3	2	$1^2 2^2$	*0.256*	$1^3 2^3 3^1$	*2.924*	*0.500*	*0.500*	p
1203	15	1	8	4	10	4	4	2	$1^3 2^4 4^1$	*0.273*	$3^1 4^3 5^3$	*3.623*	*0.933*	*0*	h

$|E|$ – number of edges; k – number of components; N^+ – number of edge- or pair(+)orbits; N^- – number of ”non-edge”- or pair(–)orbits; ***CL*** – largest clique; ***MC*** – largest girth; ***DM*** – diameter.

Symmetry sign: ***SEV*** – ***sign of edge symmetry***, where large numbers **N** present the power of edge orbits and small upper numbers ^{N} the number of edge orbits.

Valence sign: ***DEG*** – where large numbers **N** present the valence of edges and small upper numbers ^{N} the number of edges with valence **N.**

Measures: ***SE*** – ***value of edge symmetry***, $0 \le SE \le 1$; ***CPX*** – ***structural complexity***, that depend from the numbers of structural attributes; ***TRA*** – ***triangularity***, $0 \le TRA \le 1$; ***BRA*** – ***branching***, $0 \le BRA \le 1$; ***pro*** – special properties.

*

A main property of the structure is its symmetry and the examples are arranged by *symmetry properties* of structure. We begin at presentation the ***transitive or vertex symmetric structures***. Usually take the transitive graphs as certain cases of regular *cubic, quartic, quintic etc* graphs. All the vertices of a transitive structure belong to one vertex orbit and we call these to *vertex symmetric structures.* Among all the graphs are they very exceptional. Extreme case of they is ***complete symmetry***, which are present as *complete- and empty graphs.* Transitive or vertex symmetric structures divide by numbers of pair(–)- and pair(+)orbits to three different classes: 1) ***bisymmetric structures,*** having only one pair(+)- and one pair(–)orbit; 2) ***mono-symmetric structures,*** having one pair(+)- and several pair(–)orbit; 3) ***poly- or multi-symmetric structures,*** having several pair(+)- and several pair(–)orbit. ***Non-transitive structures*** have any vertex orbits and divide by numbers of vertex orbits to two different classes: 4) ***locally- or partially symmetric structures***, where the number of vertex orbits is less than the number of vertices; 5) ***0-symmetric structures***, where the number of vertex orbits is equal to the number of vertices.

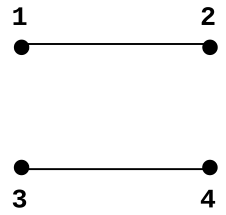
1. BISYMMETRIC GRAPHS

Bisymmetry mean in the framework of transitivity *coexistence the edge- and „non-edge” symmetry* that are very extreme cases. The complement of bisymmetric graph is also bisymmetric. They have only two adjacent structures – *an adjacent sub-structure and an adjacent super-structure*. Their number of automorphisms is large but sign matrix simple. The bisymmetric graphs we treat more profoundly.

1.1. All the bisymmetric structures with 4 to 20 vertices

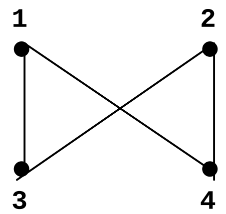
The smallest ***bisymmetric*** graph is with 4 vertices. There exist only one 4-vertices bisymmetric graph pair.

<u>**Example 1.**</u> Graph **B4-2,** its complement **B4-4** and their processing results in the form of pair signs, sign matrices with *u*-signs and corresponding measures:



A:-0.2.0; ***B:+1.2.1.***

1	2	3	4	i	AB	deg
0	**B**	-A	-A	**1**	21	1
	0	-A	–A	**2**	21	1
		0	**B**	**3**	21	1
			0	**4**	21	1



A:-2.4.4; ***B:+3.4.4.***

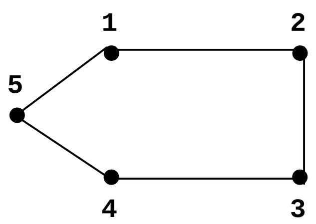
1	2	3	4	i	AB	deg
0	-A	**B**	**B**	**1**	12	2
	0	**B**	**B**	**2**	12	2
		0	-A	**3**	12	2
			0	**4**	12	2

SRV	HR	SR	aut
$2^1 4^1$	0.2764	**0.6448**	**8**

<u>Comments:</u> **a)** Graph **B4-2** consist of ***two component 2-clique*** (see pair sign ***+1.2.1***), it is ***2-clque regular***, i.e. all the vertices belong to a 2-clique. **b)** Its complement **B4-4** is ***bipartite***, where its parts correspond to the cliques of **B4-2.** In present case is **B4-4** also ***bi-clique.*** **c)** The pair sign ***+3.4.4*** of complement **B4-4** explain that it is ***4-girth regular***, i.e. all the vertices belong to a (***+d***+1=3+1=)***4-girth***.

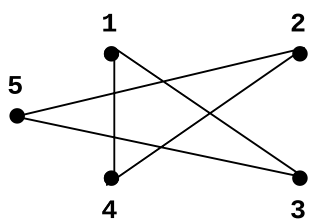
There exist only one bisymmetric graph with 5 vertices.

<u>**Example 2.**</u> Graph **B5-5,** its complement **B5-5C** and their processing results in the form of pair signs, sign matrices with *u*-signs and corresponding measures:



A:-2.3.2; ***B:+4.5.5.***

1	2	3	4	5	i	AB	deg
0	**B**	-A	-A	**B**	**1**	22	2
	0	**B**	-A	-A	**2**	22	2
		0	**B**	A	**3**	22	2
			0	**B**	**4**	22	2
				0	**5**	22	2



A:-2.3.2; ***B:+4.5.5.***

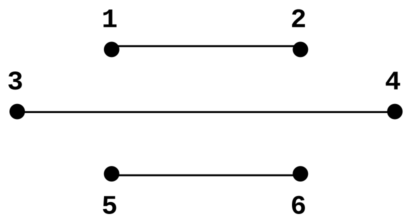
1	2	3	4	5	i	AB	deg
0	-A	**B**	**B**	-A	**1**	22	2
	0	-A	**B**	**B**	**2**	22	2
		0	-A	**B**	**3**	22	2
			0	–A	**4**	22	2
				0	**5**	22	2

SRV	*HR*	*SR*	*aut*
5^2	0.3010	**0.6990**	**10**

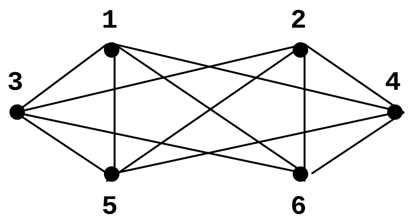
Comments: **a)** Graph **B5-5** is ***self-complemented***, i.e. its complement **B5-5C** is *isomorphic* with **B5-5** or they *structures are identical.* This expressed by identity of pair signs and equivalency of sign matrices. **b)** Pair sign ***+4.5.5*** means, that it is a 5-girth, i.e. it is ***5-girth regular***.

Among 6-vertices graphs exist there two pairs of bisymmetric graphs.

Example 3. Graph **B6-3,** its complement **B6-12** and their processing results in the form of pair signs, sign matrices with *u*-signs and corresponding measures:




```
         A:-0.2.0; B:+1.2.1.
| 1  2  3  4  5  6|  i   AB  deg
| 0  B -A -A -A -A|  1   41   1
     0  B -A -A -A|  2   41   1
        0  B -A -A|  3   41   1
           0  B -A|  4   41   1
              0  B|  5   41   1
                 0|  6   41   1
```

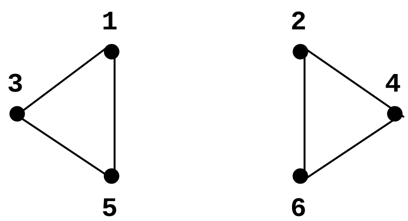



```
         A:-2.6.12; B:+2.4.5.
| 1  2  3  4  5  6|  i  AB  deg
| 0 -A  B  B  B  B|  1  14   4
     0 -A  B  B  B|  2  14   4
        0 -A  B  B|  3  14   4
           0 -A  B|  4  14   4
              0 -A|  5  14   4
                 0|  6  14   4
```

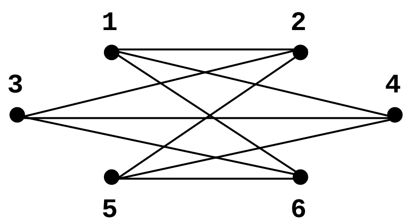
SRV	*HR*	*SR*	*aut*
$3^1 12^1$	0.2173	**0.8152**	**48**

Comments: **a)** Graph **B6-3** consist of ***three component 2-clique***, it is ***2-clique regular***. **b)** Complement **B6-12** is ***three-partite***, where its parts correspond to the 2-cliques of **B6-3**. It is a ***part-clique***, exactly ***3-part-clique*** or ***tri-clique***. **c)** From pair sign ***+2.4.5*** be conclude, that it is ***3-girth- or –clique regular***, i.e. all the vertices belong to a *triangel*.

Example 4. Graph **B6-6,** its complement **B6-9** and their processing results in the form of pair signs, sign matrices with *u*-signs and corresponding measures:




```
         A:-0.2.0; B:+2.3.3.
| 1  2  3  4  5  6|  i   ABC  deg
| 0 -A  B -A  B -A|  1   32    2
     0 -A  B -A  B|  2   32    2
        0 -A  B -A|  3   32    2
           0 -A  B|  4   32    2
              0 -A|  5   32    2
                 0|  6   32    2
```




```
         A: -2.5.6; B:+3.6.9.
| 1  2  3  4  5  6|  i   AB  deg
| 0  B -A  B -A  B|  1   23   3
     0  B -A  B -A|  2   23   3
        0  B -A  B|  3   23   3
           0  B -A|  4   23   3
              0  B|  5   23   3
                 0|  6   23   3
```

SRV	*HR*	*SR*	*aut*
$6^1 9^1$	0.2923	**0.7515**	**72**

Comments: **a)** Graph **B6-6** consist of ***two component 3-clique***, it is ***3-clique regular***. Consequently, the complement **B6-9** is ***bipartite***, where its parts correspond to 3-cliques of **B6-6**. It is also a ***bi-clique.*** **b)** Pair sign ***+3.6.9*** is a *complete invariant* and **B6-9** is ***4-girth regular***, i.e. all its vertices belong to 4-girth.

Among graphs with 7 vertices ***bisymmetric structures no exist.*** „Almost bisymmetric” is a 7-girth with its complement, they have three pair orbits and are *mono-symmetric*. Now we can they present only by sign matrices that contain all the data about structure.

Example 5. Processing results of graph **M7-7** and its complement **M7-14** in the form of pair signs, sign matrices with *u*-signs and corresponding measures:

A:-3.4.3; B:-2.3.2; ***C:+6.7.7.***

1	2	3	4	5	6	7	i	ABC	deg
0	-B	-A	**C**	**C**	-A	–B	**1**	222	2
	0	-B	-A	**C**	**C**	-A	**2**	222	2
		0	-B	-A	**C**	**C**	**3**	222	2
			0	-B	-A	**C**	**4**	222	2
				0	-B	-A	**5**	222	2
					0	–B	**6**	222	2
						0	**7**	222	2

A:-2.5.7; ***B:+2.3.3; C:+2.4.5.***

1	2	3	4	5	6	7	i	ABC	deg
0	**C**	**B**	-A	-A	**B**	**C**	**1**	222	4
	0	**C**	**B**	-A	-A	**B**	**2**	222	4
		0	**C**	**B**	-A	-A	**3**	222	4
			0	**C**	**B**	-A	**4**	222	4
				0	**C**	**B**	**5**	222	4
					0	**C**	**6**	222	4
						0	**7**	222	4

SRV	HR	SR	aut
$7^1 14^1$	0.2764	**0.7909**	**14**

Comments: **a)** From complete pair sign ***+6.7.7*** conclude, that graph **M7-7** really constitute a 7-girth. **b)** As the distances between vertices are differ *–d=2* and *–d=3*, then exist two pair(–)orbits *–A* and *–B* and the structure of graph is ***mono-***, in present case ***edge-*** *or* (+)***symmetric***. **c)** Structure of the complement **M7-14** consist of 3-girths and is also ***mono-symmetric***, exactly ***„non-edge”-*** *or* (–)***symmetric***.

Among transitive graphs with 8 vertices are bisymmetric only *2- and 4-clique-regular* structures with their complements.

Example 6. Processing results of graph **B8-4** and its complement **B8-24** in the form of pair signs, sign matrices with *u*-signs and corresponding measures:

A:-0.2.0; ***B:+1.2.1.***

1	2	3	4	5	6	7	8	i	AB	deg
0	**B**	-A	-A	-A	-A	-A	-A	**1**	61	1
	0	-A	-A	-A	-A	-A	-A	**2**	61	1
		0	**B**	-A	-A	-A	-A	**3**	61	1
			0	-A	-A	-A	–A	**4**	61	1
				0	**B**	-A	-A	**5**	61	1
					0	-A	-A	**6**	61	1
						0	**B**	**7**	61	1
							0	**8**	61	1

A:-2.8.24; ***B:+2.6.13.***

1	2	3	4	5	6	7	8	i	AB	deg
0	-A	**B**	**B**	**B**	**B**	**B**	**B**	**1**	16	6
	0	**B**	**B**	**B**	**B**	**B**	**B**	**2**	16	6
		0	-A	**B**	**B**	**B**	**B**	**3**	16	6
			0	**B**	**B**	**B**	**B**	**4**	16	6
				0	-A	**B**	**B**	**5**	16	6
					0	**B**	**B**	**6**	16	6
						0	-A	**7**	16	6
							0	**8**	16	6

SRV	HR	SR
$4^1 24^1$	0.1781	**0.8769**

Comments: **a)** Graph **B8-4** consist of ***four component 2-clique***, it is ***2-clique-regular***. Consequently, the complement **B8-24** is ***four-partite***, where its parts correspond to 2-cliques of **B8-4. b)** Complement **B8-24** constitute a ***quadro-clique*** and is ***4-clique-regular***, i.e. all its vertices belong to 4-clique.

Example 7. Processing results of graph **B8-12** and its complement **B8-16** in the form of pair signs, sign matrices with *u*-signs and corresponding measures:

A:-0.2.0; ***B:+2.4.6.*** *A:-2.6.8;* ***B:+3.8.16.***

1	2	3	4	5	6	7	8	i	AB	deg
0	B	B	B	-A	-A	-A	-A	1	43	3
	0	B	B	-A	-A	-A	-A	2	43	3
		0	B	-A	-A	-A	-A	3	43	3
			0	-A	-A	-A	-A	4	43	3
				0	B	B	B	5	43	3
					0	B	B	6	43	3
						0	B	7	43	3
							0	8	43	3

1	2	3	4	5	6	7	8	i	AB	deg
0	-A	-A	-A	B	B	B	B	1	34	4
	0	-A	-A	B	B	B	B	2	34	4
		0	-A	B	B	B	B	3	34	4
			0	B	B	B	B	4	34	4
				0	-A	-A	-A	5	34	4
					0	-A	–A	6	34	4
						0	-A	7	34	4
							0	8	34	4

SRV	HR	SR
$12^1 16^1$	0.2966	**0.7906**

Comments: **a)** Graph **B8-12** consist of ***two component 4-clique***, it is ***4-clique-regular***. Consequently, the complement **B8-16** is ***bipartite***, where its parts correspond to 4-cliques of **B8-12**. It is also a ***bi-clique.*** **b)** Pair sign ***+3.8.16*** is a *complete invariant* and **B8-16** is ***4-girth-regular***, i.e. all its vertices belong to 4-girth.

Now we can to formulate some Propositions:

Proposition 1. The ***complement*** of a graph that consist of ***r component n-clique*** (*n-clique-regular*), is ***r-partite***, where the power of parts is ***n***, i.e. it is *n-part- regular*.
Comment: This mean, that disconnected component *n-cliques* change to the *parts* of its complement, where the number ***r*** of *n*-cliques equal to number of parts and the power ***n*** of *n*-cliques equal to power of parts.

Proposition 2. The first pair sign of a graph with component cliques is ***sign of non-connectivity*** *-0.2.0* and the other is ***clique sign.***
Comment: The clique sins are ***+1.2.1*** (2-clique) or ***+2.3.3*** (3-clique) or ***+2.4.6*** (4-clique) or ***+2.5.10*** (5-clique) etc.

Proposition 3. Induced on the ground of component *n*-cliques ***connected bisymmetric structure*** constitute a ***r-clique*** that contain cliques with power ***r***, i.e. it is on ***r-clique regular***.
Comment: Bi-clique is *2-clique-regular*, tri-clique is *3-clique-regular etc.* All the r-cliques we call ***Reval cliques***.

Proposition 4. We call the ***r-cliques*** by their number of parts correspondingly to ***bi-, tri-, quadro-, quinta-, sexta-, septa-, octa-, nona-, deca-, undeca-*** etc ***-clique.***
Comments: After there presented structures exist following bisymmetric structures:

two bisymmetric structures with 9 vertices that induced by component 3-cliques correspondingly to a ***tri-clique;***

four bisymmetric structures with 10 vertices that induced by component 2- and 5-cliques correspondingly to ***quinta- and bi-cliques;***

eight bisymmetric structures with 12 vertices that induced by component 2-, 3-, 4- and 6-cliques correspondingly to ***sexta-, quadro-, tri- and bi-cliques;***

four bisymmetric structures with 14 vertices that induced by component 2-, 7-cliques correspondingly to ***septa- and bi-cliques;***

four bisymmetric structures with 15 vertices that induced by component 3-, 5-cliques correspondingly to ***quinta- and tri-cliques;***

six bisymmetric structures with 16 vertices that induced by component 2-, 4- and 8-cliques correspondingly to ***octa, quadro- and bi-cliques;***

eight bisymmetric structures with 18 vertices that induced by component 2-, 3-, 6- and 9-cliques correspondingly to ***nona-, sexta-, tri- and bi-cliques;***

eight bisymmetric structures with 20 vertices that induced by component 2-, 4-, 5- and 10-cliques correspondingly to ***deca-, quinta-, quadro- and bi-cliques;***

etc.

Proposition 5. ***Bi- or 2-partite structure is 4-girth-regular***, i.e. its pair(+)sign begin with ***+3***.

Proposition 6. The number of edges ***E*** of a ***r***-clique equal to $E=n^2r(r-1):2$
Comment: It is the lawfulness, it is simply recognized by *deg*-column of sign matrix.

Example 8. Still one example on a ***r-clique***: Processing results of graph **B18-108** as complement of **B18-45** which consist of three component 6-cliques. Their pair signs, sign matrices with *u*-signs and corresponding measures:

A:-2.14.60; ***B:+2.8.13.***

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	i	A	B	deg
0	-A	B	B	B	B	-A	-A	B	B	B	B	-A	-A	B	B	B	B	1	5	12	12
	0	B	B	B	B	-A	-A	B	B	B	B	-A	-A	B	B	B	B	2	5	12	12
		0	-A	B	B	B	B	-A	-A	B	B	B	B	-A	-A	B	B	3	5	12	12
			0	B	B	B	B	-A	-A	B	B	B	B	-A	-A	B	B	4	5	12	12
				0	-A	B	B	B	B	-A	-A	B	B	B	B	-A	-A	5	5	12	12
					0	B	B	B	B	-A	-A	B	B	B	B	-A	-A	6	5	12	12
						0	-A	B	B	B	B	-A	-A	B	B	B	B	7	5	12	12
							0	B	B	B	B	-A	-A	B	B	B	B	8	5	12	12
								0	-A	B	B	B	B	-A	-A	B	B	9	5	12	12
									0	B	B	B	B	-A	-A	B	B	10	5	12	12
										0	-A	B	B	B	B	-A	-A	11	5	12	12
											0	B	B	B	B	-A	-A	12	5	12	12
												0	-A	B	B	B	B	13	5	12	12
													0	B	B	B	B	14	5	12	12
														0	-A	B	B	15	5	12	12
															0	B	B	16	5	12	12
																0	-A	17	5	12	12
																	0	18	5	12	12

SRV	HR	SR
$45^1 108^2$	0.2631	0.8796

Comments: **a)** Graph **B18-108** is ***tri-partite*** with parts **1,2,7,8,13,14; 3,4,9,10,15,16** and **5,6,11,12,17,18** and also a ***tri-clique*** and ***3-clique regular.*** **b)** Consequently, its complement **B18-45** consist of ***three component 6-clique*** which correspond to parts of **B18-108** and is ***6-clique regular***.

1.2. Bisymmetry and strong regularity

A graph ***G*** said ***strongly regular*** with parameters (*k*,***a***,***b***) if it is a *k*-regular incomplete graph such that any two adjacent vertices have exactly ***a***≥0 common neighbours and any two non-adjacent vertices have ***b***≥1 common neighbours. Existence in bisymmetric structure exactly two pair signs, $-d.n_1.q$ and $+d.n_2.q$, mean that by ±***d***=2 *has each disadjacent vertex pair exactly* n_1 *common neighbours* and *each adjacent vertex pair* n_2 *common neighbours*.

Proposition 7. All the bisymmetric graphs are also ***strongly regular***, but no on the contrary.
Comment: The numbers n_1 and n_2 of common neighbours can be stay constant also by existence more that two pair signs, i.e. by mono-, multi- and local symmetries. Consequently, strongly regular graphs can be also *mono-, multi- and partial symmetric.*

After bisymmetric graphs that are induced by component *n*-cliques there exists also some well-known bisymmetric and strongly regular graphs.

Example 9. Processing results of graph **B9-18** and its complement **B9-18C** in the form of pair signs, sign matrices with *u*-signs and corresponding measures:

A:-2.4.4; **B:+2.3.3.**

1	2	3	4	5	6	7	8	9	i	AB	deg
0	B	B	B	B	-A	-A	-A	-A	1	44	4
	0	B	-A	-A	B	B	-A	–A	2	44	4
		0	-A	-A	-A	-A	B	B	3	44	4
			0	B	-A	B	B	–A	4	44	4
				0	B	-A	-A	B	5	44	4
					0	B	-A	B	6	44	4
						0	B	–A	7	44	4
							0	B	8	44	4
								0	9	44	4

1	2	3	4	5	6	7	8	9	i
0	-A	-A	-A	-A	B	B	B	B	1
	0	-A	B	B	-A	-A	B	B	2
		0	B	B	B	B	-A	-A	3
			0	-A	B	-A	-A	B	4
				0	-A	B	B	-A	5
					0	-A	B	-A	6
						0	-A	B	7
							0	-A	8
								0	9

SRV	*HR*	*SR*
18^2	*0.3010*	***0.8066***

<u>Comment:</u> Structure **B9-18** is ***self-complemented*** and ***3-clique-*** or ***3-girth-regular*** bisymmetric and strongly regular, which consist in ***six 3-girths*** so, that each vertex belong to two different 3-girths but each edge to one 3-girth.

<u>Example 10.</u> Processing results of graph **B10-15** and its complement **B10-30** in the form of pair signs, sign matrices with *u*-signs and all the corresponding invariants, measures and comments:

A:-2.3.2; ***B:+4.10.15.***

1	2	3	4	5	6	7	8	9	10	*i*	*AB*	*deg*
0	**B**	-A	-A	**B**	**B**	-A	-A	-A	-A	**1**	***63***	3
	0	**B**	-A	-A	-A	**B**	-A	-A	-A	**2**	***63***	3
		0	**B**	-A	-A	-A	**B**	-A	-A	**3**	***63***	3
			0	**B**	-A	-A	-A	**B**	-A	**4**	***63***	3
				0	-A	-A	-A	-A	**B**	**5**	***63***	3
					0	-A	**B**	**B**	-A	**6**	***63***	3
						0	-A	**B**	**B**	**7**	***63***	3
							0	-A	**B**	**8**	***63***	3
								0	-A	**9**	***63***	3
									0	**10**	***63***	3

A:-2.6.12; ***B:+2.5.8.***

1	2	3	4	5	6	7	8	9	10	*i*	*AB*	*deg*
0	-A	**B**	**B**	-A	-A	**B**	**B**	**B**	**B**	**1**	***36***	6
	0	-A	**B**	**B**	**B**	-A	**B**	**B**	**B**	**2**	***36***	6
		0	-A	**B**	**B**	**B**	-A	**B**	**B**	**3**	***36***	6
			0	-A	**B**	**B**	**B**	-A	**B**	**4**	***36***	6
				0	**B**	**B**	**B**	**B**	–A	**5**	***36***	6
					0	**B**	-A	-A	**B**	**6**	***36***	6
						0	**B**	-A	–A	**7**	***36***	6
							0	**B**	–A	**8**	***36***	6
								0	**B**	**9**	***36***	6
									0	**10**	***36***	6

Common invariants and measures of graph and its complement:

Symmetry	*|V|*	*|R|*	*K*	*N*	*SVV*	*SV*	*SRV*	*HR*	*SR*	*aut*
Bisymmetry	**10**	**45**	**1**	**2**	**10^1**	*1.000*	**15^{1}30^1**	*0.2764*	***0.8328***	120

Distinguishing invariants and measures:

G	*|E|*	*k*	*N$^+$*	*N$^-$*	*P*	*CL*	*MC*	*DM*	*SEV$^+$*	*SE$^+$*	*TRA*	*BRA*
B10-15	15	1	1	1	2	2	5	2	**15^1**	*1.000*	*0*	*0*
B10-30	30	1	1	1	2	4	4	2	**30^1**	*1.000*	*1.000*	*0*

<u>Comments:</u> **a)** Structure **B10-15** appears to well-known ***Petetrsen graph***. **b)** On structural aspect is Petersen graph ***unique*** and *recognizable* by its ***complete pair sign +4.10.15*** (its 10 vertices form 15 adjacent pairs that belong to *5*-girths). Another graph with such pair sign no exist. **c)** Characteristic properties of Petersen graph are ***bisymmetry, strongly-, 5-girth-, 2-distance- and 3-valence regularity***. **d)** By Graph Atlas belong Petersen graph to *regular, connected cubic graphs* (p 127, **C27)**, *symmetric cubic graphs* (p 167, with Heawood's graph), *(3,5)-cage-graphs* (p 271) and to *snark-graphs* (p 276, **Sn1**). **e)** By Graph Atlas p 263: ***Cages*** *are regular graphs of given girth with minimum vertices; specifically, a (k,g)cage is a k-regular graph of girth g with the minimum number of vertices.* **f)** Complement of Petersen graph **B10-30** is ***bisymmetric, strongly-, 4-clique-, 2-distance-*** and ***6-valence-regular***.

We know, that the number of connected, 4-valence-regular graphs is 265, among this only two transitive. Bisymmetric and strongly regular graphs with 11 vertices no exist. We were shown, that among graphs with 12 vertices are eight bisymmetric and strongly regular graph. A. Titov [1976] was fixed one bisymmetric structure with 13 vertices.

<u>Example 11.</u> Processing results of graph **B13-39** and its complement **B13-39C** in the form of pair signs, sign matrices with *u*-signs and all the corresponding invariants, measures and comments:

A:-2.5.7; ***B:+2.4.5.***

1	2	3	4	5	6	7	8	9	10	11	12	13	*i*	*AB*
0	**B**	-A	**B**	**B**	-A	-A	-A	-A	**B**	**B**	-A	**B**	**1**	***66***
	0	**B**	-A	**B**	**B**	-A	-A	-A	-A	**B**	**B**	-A	**2**	***66***
		0	**B**	-A	**B**	**B**	-A	-A	-A	-A	**B**	**B**	**3**	***66***
			0	**B**	-A	**B**	**B**	-A	-A	-A	-A	**B**	**4**	***66***
				0	**B**	-A	**B**	**B**	-A	-A	-A	–A	**5**	***66***
					0	**B**	-A	**B**	**B**	-A	-A	-A	**6**	***66***
						0	**B**	-A	**B**	**B**	-A	–A	**7**	***66***
							0	**B**	-A	**B**	**B**	-A	**8**	***66***
								0	**B**	-A	**B**	**B**	**9**	***66***
									0	**B**	-A	**B**	**10**	***66***
										0	**B**	–A	**11**	***66***
											0	**B**	**12**	***66***
												0	**13**	***66***

A:-2.5.7; ***B:+2.4.5.***

1	2	3	4	5	6	7	8	9	10	11	12	13
0	-A	**B**	-A	-A	**B**	**B**	**B**	**B**	-A	-A	**B**	–A
	0	-A	**B**	-A	-A	**B**	**B**	**B**	**B**	-A	-A	**B**
		0	-A	**B**	-A	-A	**B**	**B**	**B**	**B**	-A	–A
			0	-A	**B**	-A	-A	**B**	**B**	**B**	**B**	-A
				0	-A	**B**	-A	-A	**B**	**B**	**B**	**B**
					0	-A	**B**	-A	-A	**B**	**B**	**B**
						0	-A	**B**	-A	-A	**B**	**B**
							0	-A	**B**	-A	-A	**B**
								0	-A	**B**	-A	-A
									0	-A	**B**	-A
										0	-A	**B**
											0	-A
												0

Common invariants and measures of graph and its complement:

Symmetry	*|V|*	*|R|*	*K*	*N*	*SVV*	*SV*	*SRV*	*HR*	*SR*
Bisymmetry	**13**	**78**	**1**	**2**	**13**[1]	*1.000*	**39**[2]	*0.3010*	***0.8409***

G	*|E|*	*k*	*N⁺*	*N⁻*	*P*	*CL*	*MC*	*DM*	*SEV⁺*	*SE⁺*	*TRA*	*BRA*
B13-39, B13-39C	39	1	1	1	2	3	3	2	**39**[1]	*1.000*	*1.000*	*0*

<u>Comments:</u> From equivalence of sign matrices conclude, that structure **B13-39** is ***self-complemented***. **b) B13-39** is ***bi-symmetric, strongly-, 3-clique-*** or ***3-girth-, 2-distance-*** and ***6-valence regular***.

The next graph **B16-40** is constructed by Greenwood-Gleason as in any 3-colouring of the edges of the K_{16} without monochromatic triangles, the set of edges of each colour from this graph. It called also Clebish graph.

<u>Example 12.</u> Processing results of graph **B16-40** and its complement **B16-80** in the form of pair signs, sign matrices with *u*-signs and all the corresponding invariants, measures and comments:

A:-2.4.4; ***B:+3.10.13.***

```
   1   2   3   4   5   6   7   8   9  10  11  12  13  14  15  16|   i    AB  deg
   0   B  -A  -A   B  -A  -A   B  -A  -A  -A   B  -A  -A   B  -A|   1   105    5
       0   B  -A  -A  -A   B  -A  -A   B  -A  -A  -A   B  -A  -A|   2   105    5
           0   B  -A   B  -A  -A   B  -A  -A   B  -A  -A  -A  -A|   3   105    5
               0   B  -A  -A   B  -A  -A   B  -A  -A   B  -A  -A|   4   105    5
                   0   B  -A  -A  -A   B  -A  -A   B  -A  -A  -A|   5   105    5
                       0   B  -A  -A  -A  -A  -A  -A  -A   B   B|   6   105    5
                           0   B  -A  -A   B  -A   B  -A  -A  -A|   7   105    5
                               0   B  -A  -A  -A  -A  -A  -A   B|   8   105    5
                                   0   B  -A  -A   B  -A   B  -A|   9   105    5
                                       0   B  -A  -A  -A  -A   B|  10   105    5
                                           0   B  -A  -A   B  -A|  11   105    5
                                               0   B  -A  -A   B|  12   105    5
                                                   0   B  -A  -A|  13   105    5
                                                       0   B   B|  14   105    5
                                                           0  -A|  15   105    5
                                                               0|  16   105    5
```

A:-2.8.24; ***B:+2.8.22.***

```
   1   2   3   4   5   6   7   8   9  10  11  12  13  14  15  16|   i   A B  deg
   0  -A   B   B  -A   B   B  -A   B   B   B  -A   B   B  -A   B|   1   510   10
       0  -A   B   B   B  -A   B   B  -A   B   B   B  -A   B   B|   2   510   10
           0  -A   B  -A   B   B  -A   B   B  -A   B   B   B   B|   3   510   10
               0  -A   B   B  -A   B   B  -A   B   B  -A   B   B|   4   510   10
                   0  -A   B   B   B  -A   B   B  -A   B   B   B|   5   510   10
                       0  -A   B   B   B   B   B   B   B  -A  -A|   6   510   10
                           0  -A   B   B  -A   B  -A   B   B   B|   7   510   10
                               0  -A   B   B   B   B   B   B  -A|   8   510   10
                                   0  -A   B   B  -A   B  -A   B|   9   510   10
                                       0  -A   B   B   B   B  -A|  10   510   10
                                           0  -A   B   B  -A   B|  11   510   10
                                               0  -A   B   B  -A|  12   510   10
                                                   0  -A   B   B|  13   510   10
                                                       0  -A  -A|  14   510   10
                                                           0   B|  15   510   10
                                                               0|  16   510   10
```

Common invariants and measures of graph and its complement:

Symmetry	*|V|*	*|R|*	*K*	*N*	*SVV*	*SV*	*SRV*	*HR*	*SR*
Bisymmetry	**16**	**120**	**1**	**2**	**16**[1]	*1.000*	**40**[1]**80**[1]	*0.2762*	***0.8670***

Distinguishing invariants and measures:

G	*|E|*	*k*	*N⁺*	*N⁻*	*P*	*CL*	*MC*	*DM*	*SEV⁺*	*SE⁺*	*TRA*	*BRA*
B16-40	40	1	1	1	2	2	4	2	**40^1**	*1.000*	*0*	*0*
B16-80	80	1	1	1	2	5	3	2	**80^1**	*1.000*	*1.000*	*0*

Comments: **a)** The ***bisymmetric*** and ***strongly regular*** structure **B16-40** is correspondingly to pair(+)sign ***+3.10.13*** (complete invariant of pair graph) ***4-girth regular***, that mean *partiting.* This appear to ***4-partite*** with incompletely connected parts on *4-elementical bases*. **b)** It is no quadroclique. **c)** The parts are *variety*, where, for example one variant is **A=5,8,12,15; B=3,7,10,14**; **C=1,4,9,16**; and **D=2,6,11,13**:

	A	**B**	**C**	**D**
A	0	4	6	10
B		0	10	6
C			0	4
D				0

d) From 4-elemintic parts of **B16-40** conclude the ***4-clique regularity*** of variety cliques of complement **B16-80**. **e)** On the other hand, in case of each vertex of **B16-40** its 5 adjacent vertices no have between themselves adjacencies (edges), from which conclude also a ***5-clique-regularity*** of complement **B16-80**. We can in **B16-80** to fix 16 different 5-cliques, such as (beginning at the adjacent vertices of first vertex of **B16-40**) **2,5,8,12,15**; **1,3,7,10,14**; … to ending with **6,8,10,12,14**.

Among from B. Weisfeiler [1976] constructed *strongly regular* graphs exists also *some bisymmetric*.

Example 13. Pair- and *u*-signs with general invariants and measures of Weisfeiler's transitive strongly regular graph **B15-45** and its complement **B15-60**:

A:-2.5.6; ***B:+2.3.3. u=****8.6*   *A:-2.6.11;* ***B:+2.6.12. u=****6.8.*

SRV	*HR*	*SR*
$45^1 60^1$	*0.2966*	***0.8533***

Comments: **a)** Structure **B15-45** is ***3-clique-*** or ***–girth-regular.*** **b)** Complement **B15-60** is ***5-clique-regular***.

Example 14. Pair- and *u*-signs with general invariants and measures of Weisfeiler's transitive strongly regular graph **B16-48** and its complement **B16-72**:

A:-2.4.4; ***B:+2.4.6. u=****9.6*   *A:-2.8.18;* ***B:+2.6.11. u=****6.9.*

SRV	*HR*	*SR*
$48^1 72^1$	*0.2923*	***0.8594***

Comment: Structure **B16-48** and its complement **B16-72** are ***4-clique regular***.

Example 15. Pair- and *u*-signs with general invariants and measures of Weisfeiler's transitive strongly regular graph **B17-68** and its complement **B17-68C**:

A:-2.6.11; ***B:+2.5.7. u=****8.8*   *A:-2.6.11;* ***B:+2.5.7. u=****8.8.*

SRV	*HR*	*SR*
68^2	*0.3010*	***0.8589***

Comment: Structure **B17-68** is ***self-complemented*** and ***3-clique-*** or ***3-girth-regular.***

Some bisymmetric structures with more than 20 vertices.

Example 16. Pair- and *u*-signs with general invariants and measures of Weisfeiler's transitive strongly regular graph **B21-105** and its complement **B21-105C**:

A:-2.6.12; ***B:+2.7.17. u=****10.10*   *A:-2.8.15;* ***B:+2.5.7. u=****10.10.*

SRV	*HR*	*SR*
105^2	*0.3010*	***0.8704***

Comment: Structure **B21-105** is ***6-clique-regular***, its complement **B21-105C** is ***3-clique-regular***.

Example 17. Pair- and *u*-signs with general invariants and measures of Weisfeiler's transitive strongly regular graph **B25-100** and its complement **B25-200**:

A:-2.4.4; ***B:+2.5.10. u=****16.8*   *A:-2.14.60;* ***B:+2.11.37. u=****8.***16.**

SRV	HR	SR
$100^1 200^1$	0.2764	**0.8884**

Comment: Structure **B25-100** and its complement **B25-200** are ***5-clique-regular***.

Example 18. Pair- and *u*-signs with general invariants and measures of transitive strongly regular *Paulus' graph* **B25-150** and its complement **B25-150C**:

A:-2.8.19; ***B:+2.7.14. u=****12.***12**   *A:-2.8.19;* ***B:+2.7.14. u=****12.***12.**

SRV	HR	SR
150^2	0.3010	**0.8785**

Comment: Structure **B25-150** (Paulus graph) is ***self-complemented*** and ***5-clique-regular.***

Between graphs with 21 and 25 vertices exist also many bisymmetric and strongly regular structures with 21, 22 and 24 vertices that are induced by structures that consist of component cliques. For example there exist *twelve bisymmetric structures with 24 vertices* that are induced by 2-, 3-, 4-, 6-, 8- and 12-cliques to *dudeca-, octa-, sexta-, quadro-, tri- and bi-cliques* correspondingly.

Example 19. Pair- and *u*-signs with general invariants and measures of transitive strongly regular *Schläfli's graph* **B27-135** and its complement **B27-216**:

A:-2.7.10; ***B:+2.3.3. u=****16.***10**   *A:-2.10.40;* ***B:+2.12.51. u=****10.***16.**

SRV	HR	SR
$135^1 216^1$	0.2894	**0.8863**

Comment: Structure **B27-135** is ***3-clique-regular***, its complement **B27-216** is ***clique-regular***.

More larges, with 40 vertices, bisymmetric and strongly regular structures are constructed by Netshepurenko et al [1990].

Example 20. Common pair- and *u*-signs with general invariants and measures of transitive strongly regular *Netshepurenko's graphs* **B40A-240** and **B40B-240**:

–A:–2.6.8; ***+B:+2.4.6.*** *u=27.***12**   *–A:–2.20.144;* ***+B:+2.20.142. u=****12.***27.**

SRV	HR	SR
$240^1 540^1$	0.2681	**0.9073**

Comments: **a)** Identity of characteristics of structures **B40A-240** and **B40B-240** mean their ***coincidence of symmetry properties***. **b)** The structures are ***4-clique regular***, where 4-cliques are connected. **c)** There can be *rare 4-partite* structures with 10-element parts, which is *not quadro-cliques*, that must contain 600 edges, but there exist only 240. **d)** **B40A-240** and **B40B-240** are constructed for *isomorphism testing* that on structural aspect take place by *local sign matrices* of second degree pair graphs.

*

Therefore, bisymmetric structures must be also strongly regular. In Graph Atlas [1998] exist among 70 transitive structures with to 20 vertices only 5 bisymmetric.

As we seen, are the *conditions of bisymmetry* simples. But what these mean still?

Proposition 9. Existence only the two pair signs, pair(–)- and pair(+)sign in bisymmetric structure mean the ***identity of all its pair*(–)*structures and identity of all its pair* (+)*structures***.

Comment: If the *intersections of all the disadjacent vertices, i.e. pair(-)graphs* are **isomorphic** and the *collateral intersections of all the adjacent vertices, i.e. pair(+)graphs* are **isomorphic**, then it constitute a bisymmetric structure. Bisymmetry is more strict condition than strong regularity, where the lasts can be appear also to *mono-, multi-* or even *locally symmetric.*

So we are recognized 75 bisymmetric structures with 4 to 40 vertices, mainly on the ground of componentic cliques induced structures. The results of Petersen **B10-15,** Titov **B13-39**, Weisfeiler **B15-45** et al, Greenwood-Gleason (or Clebish) **B16-40**, Paulus **B25-150**, Schläfli **B27-135** and Netshepurenko et al **B40-240** in the realm of bisymmetry are random coincides, because the first be interested on valence-regularity, other on self-complementary, third on strong regularity, fourth on colour-conjecture, others on isomorphism testing etc.

Example 21. Conclusive table of all there treated bisymmetric and strongly regular structures.

There, **c** – *components*, **p** – *partition*, **r –** *number of components or parts*, **n** – *power of components or parts*:

Nr	Notation	*deg*	*SRV*	*SR*	Comp/part		Regularity	Commentary	Pair signs	
					r	n			Pair(-)	Pair(+)
1	**B4-2**	1	$2^1 4^1$	***0.6448***	**2c**	2	*2-clique*	-	*–0.2.0*	**+1.2.1**
2	**B4-4**	2			**2p**	2	*4-girth*	**2-bi-clique**	*–2.4.4*	**+3.4.4**
3	**B5-5**	2	5^2	***0.6990***	**1c**	5	*5-girth*	**Selfcomplem.**	*–2.3.2*	**+4.5.5**
4	**B6-3**	1	$3^1 12^1$	***0.8152***	**3c**	2	*2-clique*	-	*–0.2.0*	**+1.2.1**
5	**B6-12**	4			**3p**	2	*3-clique*	**2-tri-clique**	*–2.6.12*	**+2.4.5**
6	**B6-6**	2	$6^1 9^1$	***0.7515***	**2c**	3	*3-clique*	-	*–0.2.0*	**+2.3.3**
7	**B6-9**	3			**2p**	3	*4-girth*	**3-bi-clique**	*–2.5.6*	**+3.6.9**
8	**B8-4**	1	$4^1 24^1$	***0.8769***	**4c**	2	*2-clique*	-	*–0.2.0*	**+1.2.1**
9	**B8-24**	6			**4p**	2	*4-clique*	**2-quadro-clique**	*–2.8.24*	**+2.6.13**
10	**B8-12**	3	$12^1 16^1$	***0.7906***	**2c**	4	*4-clique*	-	*–0.2.0*	**+2.4.6**
11	**B8-16**	4			**2p**	4	*4-girth*	**4-bi-clique**	*–2.6.8*	**+3.8.16**
12	**B9-9**	2	$9^1 27^1$	***0.8431***	**3c**	3	*3-clique*	-	*–0.2.0*	**+2.3.3**
13	**B9-27**	6			**3p**	3	*3-clique*	**3-tri-clique**	*–2.8.21*	**+2.5.7**
14	**B9-18**	4	18^2	***0.8066***	**3p**	3	*3-girth*	**Selfcomplem.**	*–2.4.4*	**+2.3.3**
15	**B10-5**	1	$5^1 40^1$	***0.9084***	**5c**	2	*2-clique*	-	*–0.2.0*	**+1.2.1**
16	**B10-40**	8			**5p**	2	*5-clique*	**2-quinta-clique**	*–2.10.40*	**+2.8.25**
17	**B10-15**	3	$15^1 30^1$	***0.8328***	**1c**	10	*5-girth*	***Petersen gr.***	*-2.3.2*	**+4.10.15**
18	**B10-30**	6			**1c**	10	*4-clique*	**-**	*–2.6.12*	**+2.5.8**
19	**B10-20**	4	$20^1 25^1$	***0.8196***	**2c**	5	*5-clique*	-	*–0.2.0*	**+2.5.10**
20	**B10-25**	5			**2p**	5	*4-girth*	**5-bi-clique**	*–2.7.10*	**+3.10.25**
21	**B12-6**	1	$6^1 60^1$	***0.9273***	**6c**	2	*2-clique*	-	*–0.2.0*	**+1.2.1**
22	**B12-60**	10			**6p**	2	*6-clique*	**2-sexta-clique**	*–2.12.60*	**+2.10.41**
23	**B12-12**	2	$12^1 54^1$	***0.8868***	**4c**	3	*3-clique*	-	*–0.2.0*	**+2.3.3**
24	**B12-54**	9			**4p**	3	*4-clique*	**3-quadro-clique**	*–2.11.45*	**+2.8.22**
25	**B12-18**	3	$18^1 48^1$	***0.8601***	**3c**	4	*4-clique*	-	*–0.2.0*	**+2.4.6**
26	**B12-48**	8			**3p**	4	*3-clique*	**4-tri-clique**	*–2.10.32*	**+2.6.9**
27	**B12-30**	5	$30^1 36^1$	***0.8355***	**2c**	6	*6-clique*	-	*–0.2.0*	**+2.6.15**
28	**B12-36**	6			**2p**	6	*4-girth*	**6-bi-clique**	*–2.8.12*	**+3.12.36**
29	**B13-39**	6	39^2	***0.8409***	**1c**	13	*3-clique*	**Selfcomplem.**	*–2.5.7*	**+2.4.5**
30	**B14-7**	1	$7^1 84^1$	***0.9399***	**7c**	2	*2-clique*	-	*–0.2.0*	**+1.2.1**
31	**B14-84**	12			**7p**	2	*7-clique*	**2-septa-clique**	*–2.14.84*	**+2.12.61**
32	**B14-42**	6	$42^1 49^1$	***0.8470***	**2c**	7	*7-clique*	-	*–0.2.0*	**+2.7.21**
33	**B14-49**	7			**2p**	7	*4-girth*	**7-bi-clique**	*–2.9.14*	**+3.14.49**
34	**B15-15**	2	$15^1 90^1$	***0.9119***	**5c**	3	*3-clique*	-	*–0.2.0*	**+2.3.3**
35	**B15-90**	12			**5p**	3	*5-clique*	**3-quinta-clique**	*–2.14.78*	**+2.11.46**
36	**B15-30**	4	$30^1 75^1$	***0.8711***	**3c**	5	*5-clique*	-	*–0.2.0*	**+2.5.10**
37	**B15-75**	10			**3p**	5	*3-clique*	**5-tri-clique**	*–2.12.45*	**+2.7.11**
38	**B15-45**	6	$45^1 60^1$	***0.8533***	**1c**	15	*3-clique*	***Weisfeiler***	*–2.5.6*	**+2.3.3**
39	**B15-60**	8			**1c**	15	*5-clique*	**-**	*–2.6.11*	**+2.6.12**
40	**B16-8**	1	$8^1 112^1$	***0.9488***	**8c**	2	*2-clique*	-	*–0.2.0*	**+1.2.1**
41	**B16-112**	14			**8p**	2	*8-clique*	**2-octa-clique**	*–2.16.112*	**+2.14.85**

42	**B16-24**	3	**$24^1 96^1$**	***0.8955***	**4c**	4	*4-clique*	**-**	*–0.2.0*	**+2.4.6**
43	**B16-96**	12			**4p**	4	*4-clique*	***4-quadro-clique***	*–2.14.78*	**+2.10.33**
44	**B16-40**	5	**$40^1 80^1$**	***0.8670***	**4p**	4	*4-girth*	***Greenwood***	*–2.4.4*	**+3.10.13**
45	**B16-80**	10			**1c**	16	*5-clique*	**-**	*–2.8.24*	**+2.8.22**
46	**B16-48**	6	**$48^1 72^1$**	***0.8594***	**1c**	16	*4-clique*	***Weisfeiler***	*–2.4.4*	**+2.4.6**
47	**B16-72**	9			**1c**	16	*4-clique*	**-**	*–2.8.18*	**+2.6.11**
48	**B16-56**	7	**$56^1 64^1$**	***0.8557***	**2c**	8	*8-clique*	**-**	*–0.2.0*	**+2.8.28**
49	**B16-64**	8			**2p**	8	*4-girth*	***8-bi-clique***	*–2.10.10*	**+3.16.64**
50	**B17-68**	8	**68^2**	***0.8589***	**1c**	17	*3-clique*	***Selfcomplem.***	*–2.6.11*	**+2.5.7**
51	**B18-9**	1	**$9^1 144^1$**	***0.9555***	**9c**	2	*2-clique*	**-**	*–0.2.0*	**+1.2.1**
52	**B18-144**	16			**9p**	2	*9-clique*	***2-nona-clique***	*–2.18.144*	**+2.16.113**
53	**B18-18**	2	**$18^1 135^1$**	***0.9280***	**6c**	3	*3-clique*	**-**	*–0.2.0*	**+2.3.3**
54	**B18-135**	15			**6p**	3	*6-clique*	***3-sexta-clique***	*–2.17.120*	**+2.14.79**
55	**B18-45**	5	**$45^1 108^1$**	***0.8796***	**3c**	6	*6-clique*	**-**	*–0.2.0*	**+2.6.15**
56	**B18-108**	12			**3p**	6	*3-clique*	***6-tri-clique***	*–2.14.60*	**+2.8.13**
57	**B18-72**	8	**$72^1 81^1$**	***0.8626***	**2c**	9	*9-clique*	**-**	*–0.2.0*	**+2.9.36**
58	**B18-81**	9			**2p**	9	*4-girth*	***9-bi-clique***	*–2.11.18*	**+3.18.81**
59	**B20-10**	1	**$10^1 180^1$**	***0.9607***	**10c**	2	*2-clique*	**-**	*–0.2.0*	**+1.2.1**
60	**B20-180**	18			**10p**	2	*10-clique*	***2-deca-clique***	*–2.20.180*	**+2.18.45**
61	**B20-30**	3	**$30^1 160^1$**	***0.9169***	**5c**	4	*4-clique*	**-**	*–0.2.0*	**+2.4.6**
62	**B20-160**	16			**5p**	4	*5-clique*	***4-quinta-clique***	*–2.18.128*	**+2.14.73**
63	**B20-40**	4	**$40^1 150^1$**	***0.9019***	**4c**	5	*5-clique*	**-**	*–0.2.0*	**+2.5.10**
64	**B20-150**	15			**4p**	5	*4-clique*	***5-quadro-klikk***	*–2.17.105*	**+2.12.46**
65	**B20-90**	9	**$90^1 100^1$**	***0.8682***	**2c**	10	*10-clique*	**-**	*–0.2.0*	**+2.10.45**
66	**B20-100**	10			**2p**	10	*4-girth*	***10-bi-clique***	*–2.12.20*	**+3.20.100**
67	**B21-105**	10	**105^2**	***0.8704***	**1c**	21	*6-clique*	***Weisfeiler***	*–2.6.12*	**+2.7.17**
68	**B21-105**	10			**1c**	21	*3-clique*	**-**	*–2.8.15*	**+2.5.7**
69	**B25-100**	8	**$100^1 200^1$**	***0.8884***	**1c**	25	*5-clique*	***Weisfeiler***	*–2.4.4*	**+2.5.10**
70	**B25-200**	16			**1c**	25	*5-clique*	**-**	*–2.14.60*	**+2.11.37**
71	**B25-150**	12	**150^2**	***0.8785***	**1c**	25	*5-clique*	***Selfcomplem.***	*–2.8.19*	**+2.7.14**
72	**B27-135**	10	**$135^1 216^1$**	***0.8863***	**1c**	27	*3-clique*	***Schläfli***	*–2.7.10*	**+2.3.3**
73	**B27-216**	16			**1c**	27	*clique*	**-**	*–2.10.40*	**+2.12.51**
74	**B40-240**	12	**$240^1 540^1$**	***0.9073***	**1c**	40	*4-clique*	***Netshepurenko***	*–2.6.8*	**+2.4.6**
75	**B40-540**	27			**1c**	40	*clique*	**-**	*–2.20.144*	**+2.20.142**

Comments: **a)** All the presented graph-structures are *bisymmetric* and also *strongly regular*. **b)** Underlined binary signs are ***complete invariants*** of corresponding structures, these embrace all the vertices and vertex pairs of the structure and characterize only they.

Proposition 10. Pair(+)sign of *bi-clique* and pair(–)sign of *2-r-clique* are ***complete invariats*** of structure.

Materials about strongly regular graphs be find in the internet sufficiently. Become evident, that the „strong-regularics" no are on unanimity. The lists of strongly regular graphs are only partially coincided, these are short.

An excerpt from a traditional lists of strongly regular graphs (http//poeple.csse.uwa.edu.au/gordon/ remote/srgs/):

Nr	Parameters	Our nr.	Our register
1	(5, 2, 0, 1)	**3**	**B5-5**
2	(9, 4, 1, 2)	**14**	**B9-18**
3	(10, 3, 0, 1)	**17**	**B10-15**
4	(13, 6, 2, 3)	**29**	**B13-39**
5	(15, 6, 1, 3)	**38**	**B15-45**
6	(16, 5, 0, 2)	**44**	**B16-40**
7	(16, 6, 2, 2)	**46**	**B16-48**
8	(17, 8, 3, 4)	**50**	**B17-68**
9	(21, 10, 3, 6)	**67**	**B21-105**
10	(21, 10, 5, 4)	**mono**	**M21-105**
11	(25, 8, 3, 2)	**69**	**B25-100**
12	(25, 12, 5, 6)	**71**	**B25-150**

13	(26, 10, 3, 4)	**-**	**-**
14	(27, 10, 1, 5)	**72**	**B27-135**
15	(40, 12, 2, 4)	**74**	**B40-240**

A question. Where stay the strongly regular and bisymmetric ***r***-cliques from our list!?

They were presented also structures to 999 vertices, though the lasts are only one:

16	(999, 448, 172, 224)	-	-

Example 22. We can simply to induce some bisymmetric and strongly regular graphs with 999 vertices:

Nr	Notation	*deg*	|*E*|	*SR*	Regularity	Commentary	(+)signs
1	**B999-2**	2	999		*3-clique*	333 componentical 3-cliques	***+2.3.3***
2	**B999-996**	996	497502	***0.9989***	*333-clique*	333 3-elementic parts ***3-tricent-triginta-tri-clique***	**?**
3	**B999-8**	8	3996		*9-clique*	111 componentical 9-cliques	***+2.9.36***
4	**B999-990**	990	494505	***0.9979***	*111-clique*	111 9-elementic parts ***9-cent-undeca-clique***	**?**
5	**B999-110**	110	54945		*111-clique*	9 componentical 111-cliques	***+2.111.6105***
6	**B999-888**	888	443556	***0.9736***	*9-clique*	9 111-elementic parts ***111-nona-clique***	**?**
7	**B999-332**	332	165832		*333-clique*	3 componentical 333-cliques	***+2.333.55278***
8	**B999-666**	666	332667	***0.9515***	*3-clique*	3 333-elementic parts ***333-tri-clique***	**?**

Comments: **a)** The names of ***r***-cliques can be surely to criticize, but others I no find. **b)** All the ***r***-cliques are correct and connected graphs.

A conclusion. ***Bisymmetry to distinguish oneself*** from mono- and multi-symmetry or transitivity ***by its notable symmetry properties.***

2. GRAPHS WITH A SMALLER SYMMETRY

Symmetry of the vertex transitive structure depends from number of pair orbits. More than two pair orbits have ***mono-*** and ***poly-symmetric structures.*** There we can to demonstrate also on *orbit structures* and *adjacent structures*.

Proposition 11. ***Structure***, whose edges correspond to *pair signs of a fixed orbit* ΩR_n of the structure is a ***orbit structure*** G_n. Orbit structure by pair(+)signs is a partial structure of the structure, orbit structure by pair(–)signs is a partial structure of the complement.
Comments: **a)** The number of possible orbit structures equal to the number of pair orbits. **b)** The *high degree orbit structures* are orbit structures of an orbit structure. **c)** An orbit structure can be coincides with its initial structure.

On the structural aspect are *elementary changes* of graph structure expressed in the form of a *greatest subgraph* G^{sub}_{max}, obtainable by removing a connection G-e_{ij} and/or in the form of a *smallest supergraph* G^{upp}_{min}, obtainable by adding a connection G+e_{ij}. By def 2 we call greatest subgraph to ***adjacent subgraph*** G^{low} and smallest supergraph to ***adjacent supergraph*** G^{upp}. Correspondingly to conception 7 represent the adjacent graphs, obtained by strict disjunctive removing or adding all the edges of pair orbit ΩR_n an ***adjacent structure*** G^{adj}_n.

Proposition 12. For each pair orbit ΩR_n correspond just one *adjacent structure* GS^{adj}_n, where to pair(+)orbit ΩR_n^+ correspond an ***adjacent substructure*** GS^{low}_n and to pair(–)orbit, ΩR_n^- an ***adjacent superstructure*** GS^{upp}_n.
Comments: **a)** The number *N* of adjacent structures equal to the number of pair orbits of a graph. **b)** The *transition- or morphism probability* PF_n of initial structure to corresponding adjacent structure depend from the power of pair orbits and the sum of adjacent and/or disadjacent vertex pairs in structure.

In case of connected *n*-clique regular bisymmetric structures are the concrete partial *n*-cliques indeterminacy. In case of no bisymmetric structures are the concrete cliques recognizable by its *clique signs.* In any cases can be clique signs exist in *implicit* form, i.e. there exist pair signs, which are similar to clique signs.

Proposition 13. If in the sign matrix ***W*** there exist *implicit clique signs* then for recognition the cliques must be open the *local sign matrices* $\boldsymbol{W_{ij}}$ of pair graphs $\boldsymbol{g_{ij}}$ of corresponding similar pair signs $\boldsymbol{+dnq_{ij}}$.
Comment: As a rule, are the clique signs in a local sign matrix $\boldsymbol{W_{ij}}$ expressed in explicit form.

2.1. Mono-symmetric structures

Mono-symmetric structures have one pair(+)- and several pair(–)orbit and on the contrary. If mono-symmetric structure has one pair(+)orbit, then we call it (+)- or *edge-symmetric* where its complement has one pair(–)orbit and we call it (–)- or *"non-edge"-symmetric.* Usually no differentiate mono-symmetry at bisymmetry and these together both are treated as edge transitive graphs.

Example 23. Processing results of mono-symmetric graph **M14-21** and its complement **M14-70** in the form of pair signs, sign matrices with *u*-signs and all the corresponding invariants, measures and comments:

A:-3.8.9; B:-2.3.2; ***C:+5.14.21.***

1	2	3	4	5	6	7	8	9	10	11	12	13	14	i	ABC	deg
0	**C**	-B	-A	-B	**C**	-B	-A	-B	-A	-B	-A	-B	**C**	**1**	463	3
	0	**C**	-B	-A	-B	-A	-B	-A	-B	**C**	-B	-A	-B	**2**	463	3
		0	**C**	-B	-A	-B	**C**	-B	-A	-B	-A	-B	–A	**3**	463	3
			0	**C**	-B	-A	-B	-A	-B	-A	-B	**C**	–B	**4**	463	3
				0	**C**	-B	-A	-B	**C**	-B	-A	-B	–A	**5**	463	3
					0	**C**	-B	-A	-B	-A	-B	-A	–B	**6**	463	3
						0	**C**	-B	-A	-B	**C**	-B	–A	**7**	463	3
							0	**C**	-B	-A	-B	-A	–B	**8**	463	3
								0	**C**	-B	-A	-B	**C**	**9**	463	3
									0	**C**	-B	-A	–B	**10**	463	3
										0	**C**	-B	–A	**11**	463	3
											0	**C**	–B	**12**	463	3
												0	**C**	**13**	463	3
													0	**14**	463	3

A:-2.10.36; ***B:+2.8.22; C:+2.9.30.***

1	2	3	4	5	6	7	8	9	10	11	12	13	14	i	ABC	deg
0	-A	**C**	**B**	**C**	-A	**C**	**B**	**C**	**B**	**C**	**B**	**C**	–A	**1**	346	10
	0	-A	**C**	**B**	**C**	**B**	**C**	**B**	**C**	-A	**C**	**B**	**C**	**2**	346	10
		0	-A	**C**	**B**	**C**	-A	**C**	**B**	**C**	**B**	**C**	**B**	**3**	346	10
			0	-A	**C**	**B**	**C**	**B**	**C**	**B**	**C**	-A	**C**	**4**	346	10
				0	-A	**C**	**B**	**C**	-A	**C**	**B**	**C**	**B**	**5**	346	10
					0	-A	**C**	**B**	**C**	**B**	**C**	**B**	**C**	**6**	346	10
						0	-A	**C**	**B**	**C**	-A	**C**	**B**	**7**	346	10
							0	-A	**C**	**B**	**C**	**B**	**C**	**8**	346	10
								0	-A	**C**	**B**	**C**	–A	**9**	346	10
									0	-A	**C**	**B**	**C**	**10**	346	10
										0	-A	**C**	**B**	**11**	346	10
											0	-A	**C**	**12**	346	10
												0	–A	**13**	346	10
													0	**14**	346	10

Common invariants and measures of graph and its complement:

Symmetry	|V|	|R|	K	N	SVV	SV	SRV	HR	SR	aut
Mono-symmetry	**14**	**91**	**1**	**3**	**14^1**	1.000	**$21^1 28^1 42^1$**	0.4594	**0.7655**	336

Distinguishing invariants and measures:

G	|E|	k	N^+	N^-	P	CL	MC	DM	SEV^+	SE^+	TRA	BRA
M14-21	21	1	1	2	3	2	6	3	**21^1**	1.000	0	0
M14-70	70	1	2	1	3	7	3	2	**$28^1 42^1$**	0.7935	1.000	0

Comments: **a)** Structure **M14-21** appears to well-known ***Heawood graph***. **b)** On structural aspect is Heawood graph ***unique*** and *recognizable* by its ***complete pair sign +5.14.21*** (its 14 vertices form 21 adjacent pairs that belong to *6*-girths). Other graph with such pair sign no exist. **c)** Characteristic properties of Heawood graph are (+)***symmetry, 6-girth-, 2-, 3-distance-*** and ***3-valence regularity***. **d)** From 6-girth regularity conclude partitionig, it appear to ***bi-partite*** where its parts in present case divide to vertices with even numbers and vertices with odd numbers. **e)** By Graph Atlas belong Heawood graph to *regular, connected cubic-* (p 144, **C621**), *symmetric cubic-* (p 167, with Petersen graph) and also to *(3,6)-cage* graphs (p 271). **f)** Complement of Heawood graph **M14-70** is (–)***symmetric, 7-clique-, 2-distance-*** and ***10-valence regular*** where the connected *7*-cliques correspond to parts of **M14-21. g)** Heawood graph **M14-21** and its complement **M14-70** divide to three ***orbit structures*** (by orbits *A, B* and *C* correspondingly). Orbit-structure by orbit *–A* of Heawood graph (with pair signs *–A:–3.10.16; –B:–2.2.4,* ***C:+3.8.10***) is also ***bipartite*** and coincide with the orbit-structure by orbit ***B*** of complement **M14-70**. Orbit-structure by *–B* of Heawood graph (with pair signs *–A:–0.2.0,* ***B:+2.7.21***) is ***bisymmetric***, consist of ***two 7-clique components*** and coincide with orbit-structure by ***C*** of complement **M14-70**. **h)** Heawood graph has 13x14:2=91 ***adjacent graphs***, among this 21 ***adjacent sub-*** and 70 ***adjecent super-graphs.*** As the isomorphic graphs express one structure, then has Heawood graph one ***adjacent sub-structure*** with morphism probability ***PF****=21/21=1* and two ***adjacent super-structures*** with morphism probabilities ***PF****=28/70=2/5* and ***PF****=42/70=3/5* correspondingly.

Example 24. Pair- and *u*-signs with general invariants and measures of Weisfeiler's ***transitive strongly regular*** but ***mono-symmetric*** graph **M16-48** and its complement **M16-72**:

A:-2.4.5; B:-2.4.4; ***B:+2.4.5.*** *A:-2.8.19;* ***B:+2.6.10; C:+2.6.11.***

u*=6.3.6* ***u****=6.6.3.*

SRV	HR	SR
$24^1 48^2$	0.4581	**0.7796**

Comments: **a)** ***Orbit-structure*** by *–A* of **M16-48** is *isomorphic* with structure self and ***orbit-structure*** by *–B* is ***bisymmetric*** and *isomorphic* with our structure **B16-24** (see nr. 42 in table). **b)** Graph **M16-48** is ***3-clique-, 2-distance-*** and ***6-valence regular.*** **c)** Complement **M16-72** is ***4-clique-, 2-distance-*** and ***9-valence regular***.

Example 25. Processing results of mono-symmetric graph **M16-32** and its complement **M16-88** in the form of pair signs, sign matrices with *u*-signs and all the corresponding invariants, measures and comments:

A:-4.16.32; B:-3.8.12; C:-2.4.4; ***D:+3.8.10.***

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	i	ABCD	deg
0	D	-C	-B	-A	-B	-C	D	-C	D	-C	-B	-C	-B	-C	D	1	1464	4
	0	D	-C	-B	-A	-B	-C	D	-C	D	-C	-B	-C	-B	–C	2	1464	4
		0	D	-C	-B	-A	-B	-C	D	-C	D	-C	-B	-C	–B	3	1464	4
			0	D	-C	-B	-A	-B	-C	D	-C	D	-C	-B	–C	4	1464	4
				0	D	-C	-B	-C	-B	-C	D	-C	D	-C	–B	5	1464	4
					0	D	-C	-B	-C	-B	-C	D	-C	D	–C	6	1464	4
						0	D	-C	-B	-C	-B	-C	D	-C	D	7	1464	4
							0	D	-C	-B	-C	-B	-C	D	–C	8	1464	4
								0	-B	-C	D	-A	D	-C	–B	9	1464	4
									0	-B	-C	D	-A	D	–C	10	1464	4
										0	-B	-C	D	-A	D	11	1464	4
											0	-B	-C	D	–A	12	1464	4
												0	-B	-C	D	13	1464	4
													0	-B	–C	14	1464	4
														0	–B	15	1464	4
															0	16	1464	4

A:-2.10.34; ***B:+2.8.22; C:+2.8.28; D:+2.10.37.***

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	i	ABCD	deg
0	-A	D	B	C	B	D	-A	D	-A	D	B	D	B	D	-A	1	4416	11
	0	-A	D	B	C	B	D	-A	D	-A	D	B	D	B	D	2	4416	11
		0	-A	D	B	C	B	D	-A	D	-A	D	B	D	B	3	4416	11
			0	-A	D	B	C	B	D	-A	D	-A	D	B	D	4	4416	11
				0	-A	D	B	D	B	D	-A	D	-A	D	B	5	4416	11
					0	-A	D	B	D	B	D	-A	D	-A	D	6	4416	11
						0	-A	D	B	D	B	D	-A	D	-A	7	4416	11
							0	-A	D	B	D	B	D	-A	D	8	4416	11
								0	B	D	-A	C	-A	D	B	9	4416	11
									0	B	D	-A	C	-A	D	10	4416	11

0	**B**	**D**	-A	**C**	–A	**11**	4416	11
	0	**B**	**D**	-A	**C**	**12**	4416	11
		0	**B**	**D**	-A	**13**	4416	11
			0	**B**	**D**	**14**	4416	11
				0	**B**	**15**	4416	11
					0	**16**	4416	11

Common invariants and measures of graph and its complement:

Symmetry	**|V|**	**|R|**	*K*	*N*	*SVV*	*SV*	*SRV*	*HR*	*SR*
Mono-symmetry	**16**	**120**	**1**	**4**	$\mathbf{16^1}$	*1.000*	$\mathbf{8^1 32^2 48^1}$	*0.5437*	***0.7385***

Distinguishing invariants and measures:

G	**|E|**	*k*	N^+	N^-	*P*	*CL*	*MC*	*DM*	SEV^+	SE^+	*TRA*	*BRA*
M16-32	32	1	1	3	4	2	4	4	$\mathbf{32^1}$	*1.000*	*0*	*0*
M16-88	88	1	3	1	4	8	3	2	$\mathbf{8^1 32^1 48^1}$	*0.8467*	*1.000*	*0*

<u>Comments:</u> **a)** Structure **M16-32** appears to known ***hypercube graph***. Hypercube is one from miscellaneous regular graphs, selected for their interesting properties. It is the four-dimensional cube, which is regular of degree 4. **b)** Characteristics properties of hypercube graph are (+)***symmetry, 4-girth-, 4-, 3-, 2-distance-*** and ***4-valence regularity***. **c)** Complement **M16-88** is (–)***symmetric, triangular, 2-distance-*** and ***11-valence regular***. **d)** From *4-girth regularity* (+***3.8.10***) conclude its partiting. It appear to ***bipartite*** where its parts in present case consists on vertices with even numbers and vertices with odd numbers. **e)** As hypercube is bipartite, but no *bi*-clique, then its complement **M16-88** consist of two connected *8*-clique, where the cliques correspond to parts of hypercube. Thus, the complement is ***8-clique-regular.*** **f)** The number *N=4* of ***pair orbits***, i.e. also the number of *orbit-* and *adjacent structures*, and their powers coincide by hypercube and its complement. **g)** Orbit by *–A* of **M16-32** correspond to orbit by ***+C*** of **M16-88**; orbit by *–B* of **M16-32** correspond to orbit +***B*** of complement; orbit by **–***C* of hypercube correspond to orbit by ***+D*** of its complement; orbit by ***+D*** of hypercube correspond to orbit by *–A* of **M16-88**. **h)** Hypercube and its complement divide to ***four orbit structures.*** *Orbit structure* by *–A* of **M16-32** with pair signs *–A:–0.2.0,* ***B:+1.2.1*** is ***bisymmetric, 2-clique regular*** and coincide with orbit structure by +***C*** of complement. *Orbit structure* by *–B* of **M16-32** with pair signs *–A:–4.16.32; –B:–3.8.12; –C:–2.4.4;* ***D:+3.8.10*** is ***isomorphic*** with hypercube self. *Orbit structure* by *–C* of **M16-32** with pair signs *–A:–2.8.24; –B: –0.2.0;* ***C:+2.6.13*** consist of ***two components*** (with even and odd numbers) is ***triangular*** and ***2-distance regular*** and appear ***isomorphic*** with orbit structure by *–D* of Möbius-Kantor graph. **i)** Hypercube has one ***adjacent sub-structure*** and three ***adjacent super-structures*** with morphism probabilities PF_D=*32/32=1* and PF_A=*8/88=1/11,* PF_B=*32/88=4/11,* PF_C=*48/88=6/11* correspondingly.

<u>**Example 26.**</u> Processing results of mono-symmetric graph **M20-30** and its complement **M20-160** in the form of pair signs, sign matrices with *u*-signs and all the corresponding invariants, measures and comments:

-A=-5.20.30; -B=-4.8.9; -C=-3.4.3; -D=-2.3.2; ***+E=+4.8.9.***

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	*i*	*ABCDE*	*deg*
0	**E**	-D	-C	-B	-C	-D	**E**	-D	-C	-B	-A	-B	-C	-D	-D	-C	-C	-D	**E**	**1**	13663	3
	0	**E**	-D	-C	-C	-D	-D	-C	-B	-A	-B	-C	-D	**-E**	-D	-C	-B	-C	–D	**2**	13663	3
		0	**E**	-D	-D	**E**	-D	-C	-C	-B	-C	-C	-D	-D	-C	-B	-A	-B	–C	**3**	13663	3
			0	**E**	-D	-D	-C	-B	-C	-C	-D	-D	**E**	-D	-C	-C	-B	-A	–B	**4**	13663	3
				0	**E**	-D	-C	-C	-D	-D	**E**	-D	-D	-C	-B	-C	-C	-B	–A	**5**	13663	3
					0	**E**	-D	-D	**E**	-D	-D	-C	-C	-B	-A	-B	-C	-C	–B	**6**	13663	3
						0	**E**	-D	-D	-C	-C	-B	-C	-C	-B	-A	-B	-C	–C	**7**	13663	3
							0	**E**	-D	-C	-B	-A	-B	-C	-C	-B	-C	-D	–D	**8**	13663	3
								0	**E**	-D	-C	-B	-A	-B	-C	-C	-D	**E**	–D	**9**	13663	3
									0	**E**	-D	-C	-B	-A	-B	-C	-D	-D	–C	**10**	13663	3
										0	**E**	-D	-C	-B	-C	-D	**E**	-D	–C	**11**	13663	3
											0	**E**	-D	-C	-C	-D	-D	-C	–B	**12**	13663	3
												0	**E**	-D	-D	**E**	-D	-C	–C	**13**	13663	3
													0	**E**	-D	-D	-C	-B	–C	**14**	13663	3
														0	**E**	-D	-C	-C	–D	**15**	13663	3
															0	**E**	-D	-D	**E**	**16**	13663	3
																0	**E**	-D	–D	**17**	13663	3
																	0	**E**	–D	**18**	13663	3
																		0	**E**	**19**	13663	3
																			0	**20**	13663	3

-A=-2.16.102; ***+B=+2.14.78; +C=+2.14.79; +D=+2.15.89.***

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	*i*	*ABCCD*	*deg*
0	*-A*	***D***	***B***	***C1***	***B***	***D***	*-A*	***D***	***B***	***C1***	***C2***	***C1***	***B***	***D***	***D***	***B***	***B***	***D***	*–A*	**1**	***36316***	16
	0	*-A*	***D***	***B***	***B***	***D***	***D***	***B***	***C1***	***C2***	***C1***	***B***	***D***	*-A*	***D***	***B***	***C1***	***B***	***D***	**2**	***36316***	16
		0	*-A*	***D***	***D***	*-A*	***D***	***B***	***B***	***C1***	***B***	***B***	***D***	***D***	***B***	***C1***	***C2***	***C1***	***B***	**3**	***36316***	16
			0	*-A*	***D***	***D***	***B***	***C1***	***B***	***B***	***D***	***D***	*-A*	***D***	***B***	***B***	***C1***	***C2***	***C1***	**4**	***36316***	16
				0	*-A*	***D***	***B***	***B***	***D***	***D***	*-A*	***D***	***D***	***B***	***C1***	***B***	***B***	***C1***	***C2***	**5**	***36316***	16
					0	*-A*	***D***	***D***	*-A*	***D***	***D***	***B***	***B***	***C1***	***C2***	***C1***	***B***	***B***	***C1***	**6**	***36316***	16
						0	*-A*	***D***	***D***	***B***	***B***	***C1***	***B***	***B***	***C1***	***C2***	***C1***	***B***	***B***	**7**	***36316***	16
							0	*-A*	***D***	***B***	***C1***	***C2***	***C1***	***B***	***B***	***C1***	***B***	***D***	***D***	**8**	***36316***	16
								0	*-A*	***D***	***B***	***C1***	***C2***	***C1***	***B***	***B***	***D***	*-A*	***D***	**9**	***36316***	16
									0	*-A*	***D***	***B***	***C1***	***C2***	***C1***	***B***	***D***	***D***	***B***	**10**	***36316***	16
										0	*-A*	***D***	***B***	***C1***	***B***	***D***	*-A*	***D***	***B***	**11**	***36316***	16
											0	*-A*	***D***	***B***	***B***	***D***	***D***	***B***	***C1***	**12**	***36316***	16
												0	*-A*	***D***	***D***	*-A*	***D***	***B***	***B***	**13**	***36316***	16
													0	*-A*	***D***	***D***	***B***	***C1***	***B***	**14**	***36316***	16
														0	*-A*	***D***	***B***	***B***	***D***	**15**	***36316***	16
															0	*-A*	***D***	***D***	*–A*	**16**	***36316***	16
																0	*-A*	***D***	***D***	**17**	***36316***	16
																	0	*-A*	***D***	**18**	***36316***	16
																		0	*–A*	**19**	***36316***	16
																			0	**20**	***36316***	16

Common invariants and measures of graph and its complement:

Symmetry	*|V|*	*|R|*	*K*	*N*	*SVV*	*SV*	*SRV*	*HR*	*SR*
Mono-symmetry	**20**	**190**	**1**	**5**	$\mathbf{20^1}$	*1.000*	$\mathbf{10^1 30^2 60^2}$	*0.6366*	***0.5022***

Distinguishing invariants and measures:

G	*|E|*	*k*	*N⁺*	*N⁻*	*P*	*CL*	*MC*	*DM*	*SEV⁺*	*SE⁺*	*TRA*	*BRA*
M20-30	30	1	1	4	5	2	5	5	$\mathbf{30^1}$	*1.000*	*0*	*0*
M20-160	160	1	4	1	5	8	3	2	$\mathbf{10^1 30^1 60^2}$	*0.5590*	*1.000*	*0*

Comments: **a)** Structure **M20-30** appears to well-known ***dodecahedra graph***. **b)** Characteristic properties of dodecahedra graph are (+)***symmetry, 5-girth-, 5-, 4-, 3-, 2-distance-*** and ***3-valence regularity***. **c)** Complement **M20-160** is (–)***symmetric, 2-distance-*** and ***16-valence regular***. **d)** In the sign matrix of complement no exist explicit clique signs, but *local sign matrices* of pair graphs by ***+B:+2.14.78*** of complement contain *8*-clique signs ***+2.8.28.*** By corresponding local sign matrices $W_{1.4}$, $W_{5.9}$, $W_{3.16}$, $W_{6.13}$ and $W_{5.8}$ can be recognize all the ***partial 8-cliques of the general 8-clique*** of **M20-160**:

i=	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
I	●			●			●			●		●			●		●		●	
II		●			●		●		●		●			●			●			●
III	●		●			●			●			●		●		●		●		
IV		●		●		●		●			●		●			●			●	
V			●		●			●		●			●		●			●		●

e) Consequently, **M20-160** is ***8-clique-regular***, where all its partial *8*-cliques are ***intersected*** by edges of ***C2***:

i-j=	1-12	2-11	3-18	4-19	5-20	6-16	7-17	8-13	9-14	10-15
Partial-clique	**I**	**II**	**III**	**I**	**II**	**III**	**I**	**IV**	**II**	**I**
Partial-clique	**III**	**IV**	**V**	**IV**	**V**	**IV**	**II**	**V**	**III**	**V**

f) Number *N=5* of ***pair orbits***, i.e. also of *orbit-* and *adjacent-structures*, and their powers coincide by dodecahedra and its complement. Orbit by *–A* of **M20-30** correspond to orbit by ***+C2*** of **M20-160**; orbit by *–B* of **M20-160** correspond to orbit +***C1*** of complement; orbit by **–***C* of dodecahedra correspond to orbit by ***+B*** of its complement; orbit by *–D* of **M20-30** correspond to orbit by ***+D*** of **M20-160**; orbit by ***+E*** of dodecahedra correspond to orbit by *–A* of **M20-160**. **g)** ***Orbit structure*** by *–A* of **M20-30** with pair signs *–A:–0.2.0,* ***B:+1.2.1*** is ***bisymmetric, 2-clique regular*** and consist of ***10 components***. **h)** Pair signs of ***orbit structure*** by *–B* of dodecahedra coincide with pair signs of it self and this orbit structure is isomorphic with dodecahedra. **i)** ***Orbit structure*** by *–C* of **M20-30** with pair signs *–A:–3.14.30, –B:–2.4.4, –C:–2.3.2,* ***D:+2.4.6*** is (+)***symmetric*** and ***4-clique-*** and ***3-distance regular***. **j)** ***Orbit structure*** by *–D* of **M20-30** with pair signs *–A:–3.14.30, –B:–2.4.4,* ***D:+2.3.3*** is (+)***symmetric*** and ***3-clique-*** and ***3-distance regular***. **k)** Graph **M20-30** has 19x20**:**2=190 possible ***adjacent graphs***, among this 160 ***adjacent super-*** and 30 ***adjacent sub-graphs***. As isomorphic adjacent graphs constitute an ***adjacent structure***, then it has four ***adjacent super-structure*** and one ***adjacent sub-structure***. **l)** ***Adjacent structure*** are ***partially- or locally symmetric,*** where the ***morphism probability*** by *–A* to *adjacent* *super-structure* equal to ***PF****=10/160=1/16* and reconstruction probability to ***PF'****=1/159*; probability by *–B* to ***PF****=30/160=3/16* and reconstruction to ***PF'****=1/159*; probabilities by *–C* and *–D* to ***PF****=60/160=3/8* and reconstruction to ***PF'****=1/159.* Morphism probability by **+*E*** to corresponding ***adjacent sub-structure*** equal to ***PF****=30/30=1.*

2.2. Poly- or multi-symmetric structures

Poly- or multi-symmetric structures have several pair(+)- and several pair(–)orbit ΩR_n and are usually treated as *vertex transitive graphs.* On structural aspect have these, as a rule, more pair orbits, i.e. more pair signs, that in case of mono-symmetric structures. Thus, there can be arise need to complementary identification the vertex pairs.

In case many graphs can be obtain a matrix with "complementary pair signs" also by multiplication an adjacency matrix ***E*** of graph with itself up to certain degree E^n. To with, by exponentiation to certain degree increase the number of different "pair signs" to certain number, that stay constant.

Proposition 14. For obtaining the "complementary pair signs: 1) To form the adjacent matrix ***E.*** 2) Multiple it with itself $E\times E\times E\times\ldots=E^n$ and fix in case of each degree ***n*** the number ***p*** of received different "pair signs", which as rule enlarge. 3) If ***p*** more no enlarge, then to stop the multiplication and to fix the lasts products E^n and E^{n+1}.
Comments: **a)** The elements of last product (degree) E^n are complements for corresponding elements (pair signs) of sign matrix ***W.*** **b)** Unfortunately such "complementary pair signs" E^n no works by strongly regular and some others graphs.

Example 27. Processing results of poly-symmetric graph **P15-45** and its complement **P15-60** in the form of pair signs, sign matrices with *u*-signs and all the corresponding invariants, measures and comments:

-A=-2.6.12; -B=-2.4.5; -C=-2.3.2; ***+D=+2.4.5; +E=+2.5.7.***

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	i	ABCDE	deg
0	D	-C	-B	D	E	-A	-C	-C	-A	E	D	-B	-C	D	1	22442	6
	0	D	-C	-B	D	E	-A	-C	-C	-A	E	D	-B	–C	2	22442	6
		0	D	-C	-B	D	E	-A	-C	-C	-A	E	D	–B	3	22442	6
			0	D	-C	-B	D	E	-A	-C	-C	-A	E	D	4	22442	6
				0	D	-C	-B	D	E	-A	-C	-C	-A	E	5	22442	6
					0	D	-C	-B	-D	E	-A	-C	-C	–A	6	22442	6
						0	D	-C	-B	D	E	-A	-C	–C	7	22442	6
							0	D	-C	-B	D	E	-A	–C	8	22442	6
								0	D	-C	-B	D	E	–A	9	22442	6
									0	D	-C	-B	D	E	10	22442	6
										0	D	-C	-B	D	11	22442	6
											0	D	-C	–B	12	22442	6
												0	D	–C	13	22442	6
													0	D	14	22442	6
														0	15	22442	6

-A=-2.8.15; -B=-2.7.13; ***+C=+2.4.5; +D=+2.5.10; +E=+2.7.14.***

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	i	ABCDE	deg
0	-B	C	D	-B	-A	E	C	C	E	-A	-B	D	C	–B	1	24422	8
	0	-B	C	D	-B	-A	E	C	C	E	-A	-B	D	C	2	24422	8
		0	-B	C	D	-B	-A	E	C	C	E	-A	-B	D	3	24422	8
			0	-B	C	D	-B	-A	E	C	C	E	-A	–B	4	24422	8
				0	-B	C	D	-B	-A	E	C	C	E	–A	5	24422	8
					0	-B	C	D	-B	-A	E	C	C	E	6	24422	8
						0	-B	C	D	-B	-A	E	C	C	7	24422	8
							0	-B	C	D	-B	-A	E	C	8	24422	8
								0	-B	C	D	-B	-A	E	9	24422	8
									0	-B	C	D	-B	–A	10	24422	8
										0	-B	C	D	–B	11	24422	8
											0	-B	C	D	12	24422	8
												0	-B	C	13	24422	8
													0	–B	14	24422	8
														0	15	24422	8

Common invariants and measures of graph and its complement:

Symmetry	|*V*|	|*R*|	*K*	*N*	*SVV*	*SV*	*SRV*	*HR*	*SR*	*aut*
Poly-symmetry	**15**	**105**	**1**	**5**	**15^1**	*1.000*	**$15^3 30^2$**	*0.5523*	***0.7267***	*60*

Distinguishing invariants and measures:

G	|*E*|	*k*	*N^+*	*N^-*	*P*	*CL*	*MC*	*DM*	*SEV^+*	*SE^+*	*TRA*	*BRA*
P15-45	45	1	2	3	5	3	3	2	**$15^1 30^1$**	*0.8307*	*1.000*	*0*
P15-60	60	1	3	2	5	5	3	2	**$15^2 30^1$**	*0.7366*	*1.000*	*0*

<u>Comments:</u> **a)** Graph **P15-45** and its complement **P15-60** are ***triangular*** and ***2-distance-regular***. **b)** Complement **P15-60** has clique sign ***D=+2.5.10***, that mean existence of ***5-clique***. It appear to ***5-clique-regular***, which consist of three disjoint *partial-5-cliques*:

Nr.	Clique signs ***D***	Partial-*5*-cliques
1	*1-4, 1-13, 4-7, 7-10, 10-13*	**1,4,7,10,13**
2	*2-5, 2-14, 5-8, 8-11, 11-14*	**2,5,8,11,14**
3	*3-6, 3-15, 6-9, 9-12, 12-15*	**3,6,9,12,15**

c) Pair graphs of **P15-60**, that correspond to pair signs ***+C*** and ***+E*** contain edges, which make the partial-5-cliques *connected between themselves.* **d)** The number *N=5* of ***pair orbits***, i.e. also number of *orbit-* and *adjacent structures*, and their powers coincide by **P15-45** and **P15-60**. **e)** Orbit by *–A* of **P15-45** correspond to orbit by ***+E*** of **P15-60**; orbit by *–B* of **P15-45** correspond to orbit +***D*** of complement; orbit by ***–C*** of **P15-45** correspond to orbit by ***+C*** of its complement; orbit by +***D*** of **P15-45** correspond to orbit by *–B* of complement; orbit by ***+E*** of **P15-45** correspond to orbit by *–A* of **P15-60**. **f)** ***Orbit structure*** by *–A* of **P15-45** with pair signs *–A:–2.3.3, –B: –0.2.0,* ***C:+4.5.5*** is (+)***symmetric, 5-girth regular*** and consist of ***three component 5-girths*** and is *isomorphic* with orbit graph by *–B*. **g)** ***Orbit structure*** by *–C* of **P15-45** with pair signs *–A:– –3.10.14, –B:–2.4.4, –C: –2.3.2,* ***D:+3.6.7*** is (+)***symmetric*** and ***3-partite*** (where the parts correspond to partial cliques of **P15-45**) and is *isomorphic* with orbit graph by ***+D***. **h)** ***Orbit structure*** by ***+E*** of **P15-45** with pair signs *–A:–0.2.0,* ***B:+2.3.3*** is ***bisymmetric*** and consist of ***five 3-clique components***. **i)** Poly-symmetric **P15-45** has 14x15**:**2=105 possible ***adjacent graphs,*** i.e. to orbits by *–A* and *–B* corresponds 15 isomorphic *adjacent super-graphs*, to *–C* 30 *adjacent super-graphs*, to ***+D*** 30 isomorphic *adjacent sub-graphs* and to ***+E*** correspond 15 isomorphic *adjacent sub-graphs.* Thus, graph **P15-45** has ***3 adjacent super-*** and ***2 adjacent sub-structures***. **j)** Adjacent structures of **P15-45** are *locally symmetric,* where the ***morphism probabilities*** by *–A* ja *–B* are ***PF****=15/60=1/4* and *reconstructing probabilities* ***PF'****=1/61*; morphism probability by *–C* is ***PF****=30/60=1/2* and reconstruction probability ***PF'****=1/61*; by ***+D*** are corresponding values ***PF****=30/45=2/3* and ***PF'****= 1/44,* by ***+E*** probabilities ***PF****=15/45=1/3* and ***PF'****=1/44* correspondingly.

<u>**Example 28.**</u> Processing results of poly-symmetric graph **P24-36**. It is presented in the form of complemented pair signs, complete sign matrix with *u*-signs and all the corresponding invariants, measures and comments.

By Graph Atlas it is a regular, connected cubic graph, remarked by **Ct36** (p 162). On structural aspect it is a structure with *implicit pair signs,* i.e. its first degree pair signs no recognize all the pair orbits. For specification the pair signs there exist two ways: 1) to form by first degree pair signs corresponding *sign graphs* (as orbit structure by Proposition 2-11) and to operate complementary with their pair signs; 2) to use the multiplicative pair signs by Proposition 14.

The first degree pair signs $\boldsymbol{dnq_{ij}}$, their notations ***p***, corresponding pair signs $\boldsymbol{dnq_{ij}^{p=A}}$ and $\boldsymbol{dnq_{ij}^{p=F}}$ of *sign graphs*, their notations ***p**** and ***p*****, ordering numbers ***n*** of corresponding orbits, and, *multiplicative pair signs* $\boldsymbol{e_{ij}^{6}.e_{ij}^{7}}$ by products of adjacent matrices $\boldsymbol{E^*=E^6+E^7}$ (where *6* and *7* are the degrees of matrices).

dnq_{ij}	*p*	$dnq_{ij}^{p=-A}$	*p**	$dnq_{ij}^{p=-F}$	*p***	*n*	$e_{ij}^{6}.e_{ij}^{7}$
−5.18.23	*-A*	*+23.24.24*	*-A*	*−5.10.12*	*A1*	**1**	*0.108*
				−5.8.8	*A2*	**2**	*0.107*
−4.9.10	*-B*	*−8.9.8*	*-B*	*−4.7.7*	*B*	**3**	*42.0*
−4.8.8	*-C*	*−12.13.12*	*-C*	*−2.4.4*	*C*	**4**	*32.0*
−4.7.7	*-D*	*−10.11.10*	*-D*	*−2.3.2*	*D*	**5**	*33.0*
−3.8.9	*-E*	*−11.12.11*	*-E*	*−3.10.12*	*E*	**6**	*0.243*
−3.6.6	*-F*	*−7.8.7*	*-F1*	*+3.4.4*	*F1*	**7**	*0.191*
				+5.8.10	*F2*	**8**	*0.201*
		−5.6.5	*-F2*	*+3.4.4*	*F3*	**9**	*0.173*

-3.4.3	-G	-5.6.5	-G1	-3.8.10	G1	**10**	0.150
		-3.4.3	-G2	-3.6.6	G2	**11**	0.139
				-3.4.3	G3	**12**	0.130
-2.3.2	-H	-6.7.6	-H1	-6.20.26	H1	**13**	65.0
		-4.5.4	-H2	-4.7.7	H2	**14**	75.0
		-2.3.2	-H3	-2.3.2	H3	**15**	84.0
+5.10.12	**I**	-9.10.9	**I**	-3.6.6	**I**	**16**	0.239
+5.12.15	**J**	-9.10.9	**J**	-3.4.3	**J**	**17**	0.248
+5.14.18	**K**	-11.12.11	**K**	-5.8.8	**K**	**18**	0.258

Specified sign matrix ***W****:

```
   1   2   3   4   5   6   7   8   9  10  11  12  13  14  15  16  17  18  19  20  21  22  23  24 |   i deg
   0   K  H3   E  H2   I  H1  F1   B  G1   D  G3   C  A2   D  A1   B  G2  H1  F3  H2  F2  H3   J |   1   3
       0   J  H3  F2  H2  F3  H1  G2   B  A1   D  A2   C  G3   D  G1   B  F1  H1   I  H2   E  H3 |   2   3
           0   K  H3   E  H2   I  H1  F1   B  G1   D  G3   C  A2   D  A1   B  G2  H1  F3  H2  F2 |   3   3
               0   J  H3  F2  H2  F3  H1  G2   B  A1   D  A2   C  G3   D  G1   B  F1  H1   I  H2 |   4   3
                   0   K  H3   E  H2   I  H1  F1   B  G1   D  G3   C  A2   D  A1   B  G2  H1  F3 |   5   3
                       0   J  H3  F2  H2  F3  H1  G2   B  A1   D  A2   C  G3   D  G1   B  F1  H1 |   6   3
                           0   K  H3   E  H2   I  H1  F1   B  G1   D  G3   C  A2   D  A1   B  G2 |   7   3
                               0   J  H3  F2  H2  F3  H1  G2   B  A1   D  A2   C  G3   D  G1   B |   8   3
                                   0   K  H3   E  H2   I  H1  F1   B  G1   D  G3   C  A2   D  A1 |   9   3
                                       0   J  H3  F2  H2  F3  H1  G2   B  A1   D  A2   C  G3   D |  10   3
                                           0   K  H3   E  H2   I  H1  F1   B  G1   D  G3   C  A2 |  11   3
                                               0   J  H3  F2  H2  F3  H1  G2   B  A1   D  A2   C |  12   3
                                                   0   K  H3   E  H2   I  H1  F1   B  G1   D  G3 |  13   3
                                                       0   J  H3  F2  H2  F3  H1  G2   B  A1   D |  14   3
                                                           0   K  H3   E  H2   I  H1  F1   B  G1 |  15   3
                                                               0   J  H3  F2  H2  F3  H1  G2   B |  16   3
                                                                   0   K  H3   E  H2   I  H1  F1 |  17   3
                                                                       0   J  H3  F2  H2  F3  H1 |  18   3
                                                                           0   K  H3   E  H2   I |  19   3
                                                                               0   J  H3  F2  H2 |  20   3
                                                                                   0   K  H3   E |  21   3
                                                                                       0   J  H3 |  22   3
                                                                                           0   K |  23   3
                                                                                               0 |  24   3
```

For each row of sign matrix we obtain next *u*-sign:

-A1	-A2	-B	-C	-D	-E	-F1	-F2	-F3	-G1	-G2	-G3	-H1	-H2	-H3	**I**	**J**	**K**
1	1	2	1	2	1	1	1	1	1	1	1	2	2	2	**1**	**1**	**1**

Common invariants and measures of graph **P24-36** and its complement **P24-240**:

Symmetry	|V|	|R|	K	N	SVV	SV	SRV	HR	SR
Poly-symmetry	**24**	**276**	**1**	**18**	$\mathbf{24^1}$	*1.000*	$\mathbf{12^{13}24^5}$	*1.2308*	***0.4957***

Distinguishing invariants and measures of **P24-36** and its complement **P24-240**:

G	|E|	k	N^+	N^-	P	CL	MC	DM	SEV$^+$	SE$^+$	TRA	BRA
P24-36	36	1	3	15	18	2	6	5	$\mathbf{12^3}$	*0.6934*	*0*	*0*
P24-240	240	1	15	3	18	12	3	2	$\mathbf{12^{10}24^5}$	*0.5166*	*1.000*	*0*

Comments: **a)** Graph **P24-36** is ***6-girth-, 5-, 4-, 3-, 2-distance-*** and ***3-valence regular***, its complement **P24-240** is ***triangular, 2-distance-*** and ***20-valence regular***. **b)** From *6-girth regularity* conclude its ***bipartite*** with parts, in present case, on vertices with even numbers and odd numbers. **c)** As **P24-36** is bipartite, but not *bi*-clique, then its complement **P24-240** consist of two connected *12-clique*, i.e. it is ***12-clique regular***, where its cliques correspond to parts of **P24-36**. **d)** 23x24**:**2=276 vertex pairs form 18 ***pair orbits***, among theirs 240 disadjacent vertex pairs form 15 pair(–)orbits, where orbits by *–A1, –A2, –C, –E, –F1, –F2, –F3, –G1, –G2,* and *–G3* have 12 elements and orbits by *–B, –D, –H1, –H2,* and *–H3* have 24-elements. **e)** 36 adjacent vertex pairs of **P24-36** form three pair(+)orbits, where ***+I***, ***+J*** and ***+K*** have 12 elements. **f)** The number of ***orbit-*** and ***adjacent structures*** is *N=18* and their powers coincide by **P24-36** and **P24-240**, where the orbit structures by *–A1, –A2, –C, –E, –F1, –F2, –F3, –G1, –G2, –G3,* ***I, J,*** and ***K*** of **P24-36** (with pair signs *–A:–0.2.0; +**B**:+**1.2.1***) are ***bisymmetric*** and ***2-clique regular*** and themselves ***isomorphic***. Orbit structures by

–B, –D, –H1, –H2 and *–H3* constitute various *girths*. **g)** Graph **P24-36** has 15 ***adjacent super-structures*** and 3 ***adjacent sub-structures***.

Example 29. Processing results of poly-symmetric graphs **P20A-50** and **P20B-50** in the form of initial (implicit) and complemented (explicit) pair signs, sign matrix with *u*-signs and all the invariants, measures and comments.
These similar graphs, with original notations $R_{5,4}(2.2)$ and $R_{5,4}(4)$, was constructed by Valdo Praust especially for testing the structure recognition method. These, as preceding (Example 28) have also *implicit pair signs*, where for specification necessary to use *sign graphs* or *multiplicative pair signs*. In present case used the lasts.

Common pair signs of **P20A-50** and **P20B-50**:

A:-3.8.10; B:-3.6.7; C:-2.4.4; D:-2.3.2; ***E:+2.4.6; F:+3.8.16.***

By degree 5 of adjacent matrix E^5 of **P20A-50** obtained specified pair signs and corresponding sign matrix with *u*-signs:

Initial pair signs	0	-A	-B	-C		-D	**E**		**F**
Multiplicative pair signs e^5	180	125	110	165	160	80	231	233	210
Complete notation	0	-A	-B	-C1	-C2	-D	**E1**	**E2**	**F**

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	i	ABCCDEEF	deg
0	**E2**	**E1**	**E1**	**F**	C2	C1	C1	**F**	C2	C1	C1	D	A	B	B	D	A	B	B	1	24422212	5
	0	**E1**	**E1**	C2	**F**	C1	C1	C2	**F**	C1	C1	A	D	B	B	A	D	B	B	2	24422212	5
		0	**E2**	C1	C1	**F**	C2	C1	C1	**F**	C2	B	B	D	A	B	B	D	A	3	24422212	5
			0	C1	C1	C2	**F**	C1	C1	C2	**F**	B	B	A	D	B	B	A	D	4	24422212	5
				0	**E2**	**E1**	**E1**	D	A	B	B	**F**	C2	C1	C1	A	D	B	B	5	24422212	5
					0	**E1**	**E1**	A	D	B	B	C2	**F**	C1	C1	D	A	B	B	6	24422212	5
						0	**E2**	B	B	D	A	C1	C1	**F**	C2	B	B	A	D	7	24422212	5
							0	B	B	A	D	C1	C1	C2	**F**	B	B	D	A	8	24422212	5
								0	**E2**	**E1**	**E1**	A	D	B	B	**F**	C2	C1	C1	9	24422212	5
									0	**E1**	**E1**	D	A	B	B	C2	**F**	C1	C1	10	24422212	5
										0	**E2**	B	B	A	D	C1	C1	**F**	C2	11	24422212	5
											0	B	B	D	A	C1	C1	C2	**F**	12	24422212	5
												0	**E2**	**E1**	**E1**	C2	**F**	C1	C1	13	24422212	5
													0	**E1**	**E1**	**F**	C2	C1	C1	14	24422212	5
														0	**E2**	C1	C1	C2	**F**	15	24422212	5
															0	C1	C1	**F**	C2	16	24422212	5
																0	**E2**	**E1**	**E1**	17	24422212	5
																	0	**E1**	**E1**	18	24422212	5
																		0	**E2**	19	24422212	5
																			0	20	24422212	5

By degree 7 of adjacent matrix E^7 of **P20B-50** obtained specified pair signs and corresponding sign matrix with *u*-signs:

Initial pair signs	0	-A	-B		-C			-D	**E**		**F**
Mult. Pair signs e^7	4410	3437	3276	3277	4081	4088	4011	3010	4831	4803	4445
Complete notation	0	-A	-B1	-B2	-C1	-C2	-C3	-D	**E1**	**E2**	**F**

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	i	ABBCCCDEEF	deg
0	**E1**	**E2**	**E1**	**F**	C1	C2	C3	**F**	C3	C2	C1	D	B2	B1	A	D	A	B1	B2	1	2222222212	5
	0	**E1**	**E2**	C3	**F**	C1	C2	C1	**F**	C3	C2	A	D	B2	B1	B2	D	A	B1	2	2222222212	5
		0	**E1**	C2	C3	**F**	C1	C2	C1	**F**	C3	B1	A	D	B2	B1	B2	D	A	3	2222222212	5
			0	C1	C2	C3	**F**	C3	C2	C1	**F**	B2	B1	A	D	A	B1	B2	D	4	2222222212	5
				0	**E1**	**E2**	**E1**	D	A	B1	B2	**F**	C1	C2	C3	A	D	B2	B1	5	2222222212	5
					0	**E1**	**E2**	B2	D	A	B1	C3	**F**	C1	C2	B1	A	D	B2	6	2222222212	5
						0	**E1**	B1	B2	D	A	C2	C3	**F**	C1	B2	B1	A	D	7	2222222212	5
							0	A	B1	B2	D	C1	C2	C3	**F**	D	B2	B1	A	8	2222222212	5
								0	**E1**	**E2**	**E1**	A	B1	B2	D	**F**	C3	C2	C1	9	2222222212	5
									0	**E1**	**E2**	D	A	B1	B2	C1	**F**	C3	C2	10	2222222212	5
										0	**E1**	B2	D	A	B1	C2	C1	**F**	C3	11	2222222212	5
											0	B1	B2	D	A	C3	C2	C1	**F**	12	2222222212	5
												0	**E1**	**E2**	**E1**	C3	**F**	C1	C2	13	2222222212	5
													0	**E1**	**E2**	C2	C3	**F**	C1	14	2222222212	5
														0	**E1**	C1	C2	C3	**F**	15	2222222212	5
															0	**F**	C1	C2	C3	16	2222222212	5
																0	**E1**	**E2**	**E1**	17	2222222212	5
																	0	**E1**	**E2**	18	2222222212	5
																		0	**E1**	19	2222222212	5
																			0	20	2222222212	5

Common invariants and measures:

Symmetry	$\lvert V\rvert$	$\lvert R\rvert$	$\lvert E\rvert$	*k*	N^+	*K*	*CL*	*MC*	*DM*	*SVV*	*SV*	SEV^+	SE^+	*TRA*	*BRA*
Poly-symm.	**20**	**190**	**50**	**1**	**3**	**1**	4	4	3	$\mathbf{20^1}$	*1.000*	$\mathbf{10^1 20^2}$	*0.7303*	*0.250*	*0*

Distinguishing invariants and measures:

G	*N*	*P*	*SRV*	*HR*	*SR*
P20A-50	8	8	$\mathbf{10^1 20^5 40^2}$	*0.8668*	***0.6196***
P20B-50	10	10	$\mathbf{10^1 20^9}$	*0.9936*	***0.5640***

Comments: **a)** Graph **P20A-50** differ at **P20B-50** from general symmetric properties, in present case from the number of pair(-)orbits but coincide with (+)symmetric properties. **b)** Both graphs are ***4-clique-, 4-girth-, 3-, 2-distance-*** and ***5-valence regular.*** *4*-clique regularity expressed by existence there of five *4*-cliques. **c)** Both graphs have 3 pair(+)orbits, with powers ***E1*** – 20, ***E2*** – 20 and ***F*** – 20 correspondingly. **d)** Graph **P20A-50** has 5 pair(–)orbits with powers in case by *–A, -C2, –D* – 20 elements and by *–B, –C1* – 40 elements. **e)** ***Complement*** **P20A-120** has pair signs *–A:-2.14.68, -B:-2.12.47,* ***C:+2.10.35, D:+2.10.36, E:+2.11.44, F:+2.12.48*** and is ***triangular, 5-clique-*** and ***14-valence regular***. **f)** Graph **P20B-50** has 7 pair(-)orbits with powers 20. **g)** On the ground of **d** and **e** can be make conclusions about ***orbit-*** and ***adjacent structures***. **h)** Interest can be have ***orbit structures*** with more that 2-valences. *2*-valences orbit-structures constitute only various girths and their samples. **i)** Graph **P20A-50** has orbit structures by *–B* and *–C1* that are *4*-valences.
j) Orbit structure by *–B* of **P20A-50** noted here by **M20-40**. Its processing results and comments:

A:-5.18.32; B:-4.8.12; C:-3.6.8; D:-2.6.8; E:-2.4.4; ***F:+3.8.12.***

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	*i*	*ABCDEF*	*deg*
0	-D	-A	-A	-E	-E	-C	-C	-E	-E	-C	-C	-B	-B	**F**	**F**	-B	-B	**F**	**F**	**1**	244144	4
	0	-A	-A	-E	-E	-C	-C	-E	-E	-C	-C	-B	-B	**F**	**F**	-B	-B	**F**	**F**	**2**	244144	4
		0	-D	-C	-C	-E	-E	-C	-C	-E	-E	**F**	**F**	-B	-B	**F**	**F**	-B	–B	**3**	244144	4
			0	-C	-C	-E	-E	-C	-C	-E	-E	**F**	**F**	-B	-B	**F**	**F**	-B	–B	**4**	244144	4
				0	-D	-A	-A	-B	-B	**F**	**F**	-E	-E	-C	-C	-B	-B	**F**	**F**	**5**	244144	4
					0	-A	-A	-B	-B	**F**	**F**	-E	-E	-C	-C	-B	-B	**F**	**F**	**6**	244144	4
						0	-D	**F**	**F**	-B	-B	-C	-C	-E	-E	**F**	**F**	-B	-B	**7**	244144	4
							0	**F**	**F**	-B	-B	-C	-C	-E	-E	**F**	**F**	-B	–B	**8**	244144	4
								0	-D	-A	-A	-B	-B	**F**	**F**	-E	-E	-C	–C	**9**	244144	4
									0	-A	-A	-B	-B	**F**	**F**	-E	-E	-C	–C	**10**	244144	4
										0	-D	**F**	**F**	-B	-B	-C	-C	-E	-E	**11**	244144	4
											0	**F**	**F**	-B	-B	-C	-C	-E	–E	**12**	244144	4
												0	-D	-A	-A	-E	-E	-C	-C	**13**	244144	4
													0	-A	-A	-E	-E	-C	–C	**14**	244144	4
														0	-D	-C	-C	-E	-E	**15**	244144	4
															0	-C	-C	-E	-E	**16**	244144	4
																0	-D	-A	-A	**17**	244144	4
																	0	-A	-A	**18**	244144	4
																		0	-D	**19**	244144	4
																			0	**20**	244144	4

k) Orbit structure **M20-40** is (+)***symmetric, 5-partite, 4-girth-, 5-, 4-, 3-*** and ***2-distance regular***. ***Complement*** **M20-40** is (–)***symmetric, 7-clique-*** and ***2-distance regular.*** **l)** The five parts of **M20-40** correspond to *4*-cliques of **P20A-50** and these are: **I** – with vertices **1,2,3,4; II** – **5,6,7,8; III** – **9,10,11,12; IV** – **13,14,15,16**; **V** – **17,18,19,20.** Part **I** is connected with parts IV and V; **II** – with parts III and V; **III** – with parts II and IV; **IV** – with parts I and III; **V** – with parts I and II. In principle can be the parts in pair wise added, for example parts I and II to composite part **A**, parts IV and V to composite part **B** where part III appear to part **C**. **m)** Orbit structure **M20-40** *coincide* with the corresponding orbit structure by pair(+)sign of complement **P20A-50** and appear *isomorphic* with orbit structure by *–C1* of **P20A-50**.
n) The characteristics of orbit structure **M20-40**:

$\lvert V\rvert$	$\lvert R\rvert$	$\lvert E\rvert$	*K*	*N*	N^+	*SVV*	*SV*	*SRV*	*HR*	*SR*	SEV^+	SE^+	*TRA*	*BRA*
20	**190**	**40**	**1**	**6**	**1**	$\mathbf{20^1}$	*1.000*	$\mathbf{10^1 20^2 40^4}$	*0.843*	***0.630***	$\mathbf{40^1}$	*1.000*	*0*	*0*

3. NON-TRANSITIVE GRAPHS

Almost all the graphs are non-transitive. These have any vertex orbits and divide by numbers of vertex orbits to two different classes: ***locally- or partially symmetric structures***, where the number of vertex orbits is less than the number of vertices, and, ***0-symmetric structures***, where the number of vertex orbits is equal to the number of vertices. There enlarge the role of *u*- and *s*-signs for decomposing the sign matrices to vertex orbits. Decrease the role of symmetry properties, orbit structures and regularities.

Proposition 15. Decomposing the sign matrix ***W*** begin at lexicographical decomposing the rows (and columns) by *u*-signs to classes W_k. It continued with the decomposing in each framework of W_k the rows (and columns) by *s*-signs to complementary classes W_{k*}. Repeat last up to complementary decomposing no arise.

The notation of non-transitive graphs is different.

3.1. Locally- or partially symmetric structures

In locally symmetric structures there exist symmetry properties only in framework of vertex orbits.

Proposition 16. Adjacent structures of a vertex transitive, i.e. bi-, mono- or poly-symmetric, structure are *locally symmetric.*

Example 30. Processing results the adjacent structures of well-known bi-symmetric Peteresen graph, that are presented in the form of pair signs, decomposed by *u*- and *s*-signs matrices ***W*** and all the invariants, measures and comments.

By *removing* at Petersen graph (**B10-15**) an edge *i,j*=4,5 is obtained its adjacent sub-structure **L10-14**:

A:-4.10.14; B:-3.6.6; C:-2.3.2; ***D:+4.7.8; E:+4.9.12; F:+4.10.14.***

1	**1**	**1**	**1**	**2**	**2**	**2**	**2**	**3**	**3**			***orb***	***s***	
2	6	7	8	1	3	9	10	4	5	***i***	*ABC****DEF***		123	***deg***
0	-C	**F**	–C	**E**	**E**	-C	–C	-C	–C	2	*006****021***	**1**	120	3
	0	-C	**F**	**E**	-C	**E**	–C	-C	–C	6	*006****021***	**1**	120	3
		0	–C	-C	-C	**E**	**E**	-C	–C	7	*006****021***	**1**	120	3
			0	-C	**E**	-C	**E**	-C	–C	8	*006****021***	**1**	120	3
				0	-C	-C	–C	-B	**D**	1	*015****120***	**2**	201	3
					0	-C	–C	**D**	–B	3	*015****120***	**2**	201	3
						0	–C	**D**	–B	9	*015****120***	**2**	201	3
							0	-B	**D**	10	*015****120***	**2**	201	3
								0	–A	4	*124****200***	**3**	020	2
									0	5	*124****200***	**3**	020	2

By *adding* to Petersen graph (**B10-15)** an edge *i,j*=4,6 is obtained its adjacent super-structure **L10-16**:

A:-2.4.4; B:-2.3.2; ***C:+2.3.3; D:+3.4.4; E:+4.10.16.***

1	**1**	**2**	**3**	**4**	**4**	**4**	**4**	**5**	**5**			***orb***	***s1***	***s2***	
2	10	7	9	1	3	5	8	4	6	***i***	*AB****CDE***		1234	12345	***deg***
0	–B	**E**	-B	**E**	**E**	-B	–B	-B	-B	2	*06****003***	**1**	1020	01020	3
	0	**E**	-B	-B	-B	**E**	**E**	-B	-B	10	*06****003***	**1**	1020	01020	3
		0	**E**	-B	-B	-B	–B	-B	-B	7	*06****003***	**2**	2100	20100	3
			0	-B	-B	-B	–B	**C**	**C**	9	*06****201***	**3**	1002	01002	3
				0	-B	**D**	–B	-A	**D**	1	*15****021***	**4**	1011	10011	3
					0	-B	**D**	**D**	-A	3	*15****021***	**4**	1011	10011	3
						0	–B	**D**	-A	5	*15****021***	**4**	1011	10011	3
							0	-A	**D**	8	*15****021***	**4**	1011	10011	3
								0	**C**	4	*23****220***	**5**	0121	00121	4
									0	6	*23****220***	**5**	0121	00121	4

Common invariants and measures:

Symmetry	***|V|***	***|R|***	***k***	***BRA***
Local-symmetric	**10**	**45**	**1**	*0*

Distinguishing invariants and measures:

G	|E|	K	N	SVV	SV	SRV	HR	SR	TRA
L10-14(sub)	**14**	**3**	**9**	$\mathbf{2^1 4^2}$	*0.5419*	$\mathbf{1^1 2^1 4^3 6^1 8^3}$	*0.8939*	***0.4593***	*0*
L10-16(sup)	**16**	**5**	**16**	$\mathbf{1^2 2^2 4^1}$	*0.3612*	$\mathbf{1^3 2^5 4^8}$	*1.1582*	***0.2994***	*0.188*

G	N^+	N^-	P	CL	MC	DM	SEV^+	SE
L10-14(sub)	3	6	6	2	5	4	$\mathbf{2^1 4^1 8^1}$	*0.6379*
L10-16(sup)	7	9	5	3	5	2	$\mathbf{1^2 2^3 4^2}$	*0.3437*

Comments: **a)** Exactly these same structures (**sub** and **sup**) are obtainable by operating with an arbitrary edge on Petersen graph. **b)** Adjacent sub-structure of Petersen graph has *3 vertex- and 9 pair orbits*. Its adjacent super-structure has *5 vertex- and 16 pair orbits* and its symmetry value ***SR*** is smaller. **c)** From *5-girth regularity* of Petersen graph is in **L10-14** remained 14/15 or 93%, but in **L10-16** 7/15 or 47%. The first is „more peterseniсal". **d)** ***Reverse pair orbit***, that reconstruct the Petersen graph place in partial matrix $W_{3.3}$ of **L10-14** by sign *–A*; reconstructing probability ***PF'****=1/31*. *Reverse pair orbit* of **L10-16** place in partial matrix $W_{5.5}$ in the form of sign **C**; reconstructing probability ***PF'****=1/16*. **e)** Adjacent sub-structure **L10-14** is a common adjacent super-structure of 3 initial structures and a common adjacent sub-structure of 6 initial structures. Adjacent super-structure **L10-16** is a common adjacent super-structure of 7 initial structures and common adjacent sub-structure of 9 initial structures.

Example 31. Processing results of Brinkman graph **L21-42** and its complement **L21-168**, that are presented in the form of pair signs, decomposed by *u*- and *s*-signs matrices ***W*** and all the invariants, measures and comments.

Brinkman conjectured in 1970 that for all *k*≥2 and *g*≥3 there are (*k, k, g*)-graphs, that is, *k*-chromatic *k*-regular of girth at least *g*. Brinkman graph is a (*4,4,5*)-graph (Bollobas, 1998, p 175, Fig. V, 14).

A:-3.10.12; B:-3.8.9; C:-3.6.6; D:-2.3.2; ***E:+4.13.19; F:+4.14.19; G:+4.14.21.***

```
| 1  1  1  1  1  1  1| 2  2  2  2  2  2  2| 3  3  3  3  3  3  3|                    Orb
| 2  3  9 10 17 18 21| 1  5  6 13 14 19 20| 4  7  8 11 12 15 16|  i  ABC  DEFG       deg
| 0 -D -D  G  G -D -D| G  G -D -D -D -D -C|D1 D1 D2 D2 -B -B -A|  2  121 12004     1  4
     0  G -D -D  G -D| G -D  G -D -D -C -D|D1 D2 D1 -B D2 -A -B|  3  121 12004     1  4
        0 -D -D -D  G|-D  G -D  G -C -D -D|D2 D1 -B D1 -A D2 -B|  9  121 12004     1  4
           0 -D -D  G|-D -D  G -C  G -D -D|D2 -B D1 -A D1 -B D2| 10  121 12004     1  4
              0  G -D|-D -D -C  G -D  G -D|-B D2 -A D1 -B D1 D2| 17  121 12004     1  4
                 0 -D|-D -C -D -D  G -D  G|-B -A D2 -B D1 D2 D1| 18  121 12004     1  4
                    0|-C -D -D -D -D  G  G|-A -B -B D2 D2 D1 D1| 21  121 12004     1  4
                     | 0 D2 D2 D1 D1 -B -B|-A  F  F D2 D2 D1 D1|  1  121 12022     2  4
                          0 D1 D2 -B D1 -B| F -A D2  F D1 D2 D1|  5  121 12022     2  4
                             0 -B D2 -B D1| F D2 -A D1  F D1 D2|  6  121 12022     2  4
                                0 -B D2 D1|D2  F D1 -A D1  F D2| 13  121 12022     2  4
                                   0 D1 D2|D2 D1  F D1 -A D2  F| 14  121 12022     2  4
                                      0 D2|D1 D2 D1  F D2 -A  F| 19  121 12022     2  4
                                         0|D1 D1 D2 D2  F  F -A| 20  121 12022     2  4
                                          | 0 D2 D2 D1 D1  E  E|  4  220 12220     3  4
                                               0 D1 D2  E D1  E|  7  220 12220     3  4
                                                  0  E D2  E D1|  8  220 12220     3  4
                                                     0  E D2 D1| 11  220 12220     3  4
                                                        0 D1 D2| 12  220 12220     3  4
                                                           0 D2| 15  220 12220     3  4
                                                              0| 16  220 12220     3  4
```

A:-2.15.87; ***B:+2.13.64; C:+2.13.65; D:+2.13.66; E:+2.14.74; F:+2.14.75; G:+2.14.76.***

```
| 1  1  1  1  1  1  1| 2  2  2  2  2  2  2| 3  3  3  3  3  3  3|                 Orb  s
| 2  3  9 10 17 18 21| 1  5  6 13 14 19 20| 4  7  8 11 12 15 16|  i  ABCDEFG       123 deg
| 0  E  E -A -A  F  F|-A -A  F  F  F  F  D| G  G  F  F  C  C  B|  2  4121282   1  457  16
     0 -A  E  F -A  F|-A  F -A  F  F  D  F| G  F  G  C  F  B  C|  3  4121282   1  457  16
        0  F  E  F -A| F -A  F -A  D  F  F| F  G  C  G  B  F  C|  9  4121282   1  457  16
           0  F  E -A| F  F -A  D -A  F  F| F  C  G  B  G  C  F| 10  4121282   1  457  16
              0 -A  E| F  F  D -A  F -A  F| C  F  B  G  C  G  F| 17  4121282   1  457  16
                 0  E| F  D  F  F -A  F -A| C  B  F  C  G  F  G| 18  4121282   1  457  16
                    0| D  F  F  F  F -A -A| B  C  C  F  F  G  G| 21  4121282   1  457  16
                     | 0  F  F  E  E  C  C| B -A -A  F  F  G  G|  1  4121282   2  565  16
                          0  E  F  C  E  C|-A  B  F -A  G  F  G|  5  4121282   2  565  16
                             0  C  F  C  E|-A  F  B  G -A  G  F|  6  4121282   2  565  16
```

```
 0  C  F  E| F -A  G  B  G -A  F| 13  4121282  2  565  16
    0  E  F| F  G -A  G  B  F -A| 14  4121282  2  565  16
       0  F| G  F  G -A  F  B -A| 19  4121282  2  565  16
          0| G  G  F  F -A -A  B| 20  4121282  2  565  16
           | 0  F  F  E  E -A -A|  4  4220264  3  754  16
                0  E  F -A  E -A|  7  4220264  3  754  16
                   0 -A  F -A  E|  8  4220264  3  754  16
                      0 -A  F  E| 11  4220264  3  754  16
                         0  E  F| 12  4220264  3  754  16
                            0  F| 15  4220264  3  754  16
                               0| 16  4220264  3  754  16
```

Common invariants and measures:

Symmetry	|V|	|R|	K	N	SVV	SV	SRV	HR	SR
Local-symmetric	**21**	**210**	**3**	**20**	7^3	*0.6391*	$7^{12}14^7 28^1$	*1.2564*	***0.4590***

Distinguishing invariants and measures:

G	|E|	k	N^+	N^-	P	CL	MC	DM	SEV^+	SE^+	TRA	BRA
L21-42	42	1	4	11	7	2	5	3	$7^2 14^2$	*0.6443*	*0*	*0*
L21-168	168	1	16	4	7	7	3	2	$7^{10}14^5 28^1$	*0.4812*	*1.000*	*0*

Comments: **a)** Graph **L21-42** is ***5-girth-, 2-, 3-distance-*** and ***4-valence regular***. Complement **L21-168** is ***triangular, 2-distance-*** and ***16-valence regular.*** The pair signs of **L21-42** are specified by its complement **L21-168**. **b)** ***7-clique*** of **L21-168** is expressed in sign matrix as the vertices 1,5,6,13,14,19,20 of second vertex orbit. If to remove the *7*-clique at **L21-168,** then remain a ***bi-clique*** with parts 4,7,8,11,12,15,16 and 2,3,9,10,17,18,21. It stands to reason that must be remove also the inner-parts edges in partial matrices $W_{1.1}$ and $W_{3.3}$. **c)** Data about the number and powers of ***pair orbits*** of **L21-42** and **L21-168** contain in symmetry signs ***SRV*** and SEV^+. **d)** From sign matrix can be read out, that **Bri** has ***4 adjacent sub-structures*** with morphism probabilities $PF_1=7/42=1/6$, $PF_2=14/42=1/3$, $PF_3=14/42=1/3$, and $PF_4=7/42=1/6$ correspondingly. Fixed is also existence of ***16 adjacent super-structures***.

Example 32. Processing results of a Weisfeiler graph **L25-150** and its complement **L25-150C** in the form of pair signs, decomposed by *u*- and *s*-signs sign matrix and all the corresponding invariants, measures and comments.

A:-2.8.20; B:-2.8.19; C:-2.8.18; ***D:+2.7.13; E:+2.7.14; F:+2.7.15.***

```
|1  1| 2  2| 3  3| 4  4| 5| 6  6| 7  7| 8  8| 9  9|10 10|11|12|13|14 14|15|                 orb
20 24|12 14| 1  2| 9 19| 6|10 16| 8 18| 4  7|11 17|13 15|23| 3|22|21 25| 5|   i  ABCDEF    *
 0  F| C  C| C  B| F  C| C| B  F| C  E| F  C| E  F| E  C| B| F| F| F  C| F|  20  039039    1
    0| C  C| B  C| C  F| C| F  B| E  C| C  F| F  E| C  E| B| F| F| C  F| F|  24  039039    1
     | 0  F| F  C| C  C| B| F  C| B  F| B  F| E  C| F  E| F| C| F| F  C| E|  12  039039    2
          0| C  F| C  C| B| C  F| F  B| F  B| C  E| E  F| F| C| F| C  F| E|  14  039039    2
           | 0  F| F  C| E| F  C| F  B| F  E| F  C| F  C| C| F| B| C  C| E|   1  039039    3
                0| C  F| E| C  F| B  F| E  F| C  F| C  F| C| F| B| C  C| E|   2  039039    3
                 | 0  F| E| F  C| F  E| F  C| B  F| F  C| F| B| E| C  B| C|   9  039039    4
                      0| E| C  F| E  F| C  F| F  B| C  F| F| B| E| B  C| C|  19  039039    4
                       | 0| C  C| B  B| B  B| E  E| F  F| F| F| C| F  F| C|   6  066066    5
                          | 0  B| F  E| F  E| B  E| B  E| B| B| C| E  E| C|  10  066066    6
                               0| E  F| E  F| E  B| E  B| B| B| C| E  E| C|  16  066066    6
                                | 0  C| F  B| F  B| C  C| E| C| B| C  E| E|   8  066066    7
                                     0| B  F| B  F| C  C| E| C| B| E  C| E|  18  066066    7
                                      | 0  C| C  C| B  E| C| E| E| E  B| B|   4  066066    8
                                           0| C  C| E  B| C| E| E| B  E| B|   7  066066    8
                                            | 0  B| E  F| C| B| B| E  C| F|  11  066066    9
                                                 0| F  E| C| B| B| C  E| F|  17  066066    9
                                                  | 0  B| B| C| E| B  F| B|  13  066066   10
                                                       0| B| C| E| F  B| B|  15  066066   10
                                                        | 0| E| D| F  F| E|  23  066147   11
                                                           | 0| E| F  F| D|   3  066147   12
                                                              | 0| B  B| B|  22  093174   13
                                                                 | 0  E| A|  21  147066   14
                                                                      0| A|  25  147066   14
                                                                       | 0|   5  255174   15
```

Common invariants and measures:

Symmetry	|V|	|R|	K	N	SVV	SV	SRV	HR	SR
Local-symmetric	**25**	**300**	**15**	**154**	$\mathbf{1^{5}2^{10}}$	*0.1723*	$\mathbf{1^{20}2^{128}4^{6}}$	*2.1576*	***0.1290***

Distinguishing invariants and measures:

G	|E|	k	N^{+}	N^{-}	P	CL	MC	DM	SEV^{+}	SE	TRA	BRA
L25-150	150	1	80	74	6	4	3	2	$\mathbf{1^{12}2^{67}4^{1}}$	*0.1310*	*1.000*	*0*
L25-150C	150	1	74	80	6	4	3	2	$\mathbf{1^{8}2^{61}4^{5}}$	*0.1494*	*1.000*	*0*

Comments: **a)** Graph **L25-150** [Weisfeiler,1976, p 166(1)] and its complement **L25-150C** are ***strongly regular, triangular, 4-clique-, 2-distance-*** and ***12-valence regular.*** **b)** B. Weissfeiler was an uncommon who has interested in orbits. He was constructed any strongly regular graphs, among this also *self-complemented* and *0-symmetric*, that be grounded on these same pair signs. **c)** Only with six pair signs is 25×25 sign matrix by *u*- and *s*-signs be decomposed to 15 ***vertex orbits*** and to 115 partial matrices $\mathbf{W_{ki,kj}}$. **d)** 150 "disadjacent pairs" of **L25-150** form 74 ***pair***(–)***orbits***, where –*A* form a two-element orbit, by –*B* formed 33 orbits, i.e. 4 one-elements and 29 two-elements orbits, by –*C* formed 40 orbits, among this 4 one-elements, 31 two-elements and 5 four-elements orbits. **e)** 150 adjacent pairs of **L25-150** form 80 ***pair***(+)***orbits***, where by +***D*** formed 2 one-element orbits, by +***E*** formed 32 orbits, among this 4 one-elements, 27 two-elements and 1 four-elements orbits, and by +***F*** formed 46 orbits, among this 6 one- and 40 two-elements orbits. **f)** Thus, graph **L25-150** has 80 ***adjacent sub-*** and 74 ***adjacent super-structures***, in case of **L25-150C** is it opposed. **g)** Also the *sign graphs* can be have there interest.

3.2. *0*-symmetric structures

Opening the *0*-symmetric structures have sense only in case of a concrete problem. Next *0*-symmetric graph ***0*15-83** is induced from L. Võhandu for maximum clique recognition.

Example 33. Processing results of graph ***0*15-83** in the form of pair signs, decomposed by *u*- and *s*-signs sign matrix, a local sign matrix and all the corresponding invariants, measures and comments.

(A:-2.13.66; B:-2.12.57; C:-2.12.53; D:-2.11.49; E:-2.11.46; F:-2.11.45; G:-2.11.44; H:-2.10.40; I:-2.10.38; J:-2.10.37; K:-2.10.36; L:-2.10.35; M:-2.9.31 N:-2.9.30; O:-2.9.29;)
P:+2.7.17; Q:+2.7.18;R:+2.8.24; S:+2.8.25; T:+2.9.28; U:+2.9.29; V:+2.9.30; W:+2.9.31; X:+2.9.32; Y:+2.9.34; Z:+2.9.35; AA:+2.10.37;AB:+2.10.38; AC:+2.10.39; AD:+2.10.40; AE:+2.10.41; AF:+2.10.42; AG:+2.10.43; AH:+2.11.45; AI:+2.11.46; AJ:+2.11.47; AK:+2.11.48; AL:+2.11.49; AM:+2.11.50; AN:+2.12.55; AO:+2.12.56; AP:+2.12.57; AQ:+2.12.58; AR:+2.13.64; AS:+2.13.65; AT:+2.13.66

```
| 1| 2| 3| 4| 5| 6| 7| 8| 9|10|11|12|13|14|15|          Orb
|14| 7|10| 6|12| 1| 9|15|13| 5| 3| 4|11| 2| 8|    i  deg   *
  0|AQ| M|AQ| H|AG| Z|AQ| I|AM| Y| Y|AF|AK|AK|   14   11   1
     0| G|AQ|AC|AL|AD|AT|AA|AS|AJ|AJ|AO|AR|AN|    7   13   2
        0| R| P| Q| Q| U| P| K| R| R| O| L| U|   10    8   3
           0| E|AK|AD|AS| G|AM|AC|AC|AF|AK|AN|    6   11   4
              0| R| N|AA| V| X| W| W| X|AC| F|   12   10   5
                 0|AD|AN| K|AL| E| E|AE|AE|AD|    1   11   6
                    0|AD| P|AD| J| E| D| X|AB|    9   10   7
                       0| C|AP|AI|AI|AL|AO|AN|   15   13   8
                          0| V| V| V| S| V| T|   13   10   9
                             0| B|AB|AJ|AN|AJ|    5   12  10
                                0|AH|AB|AC|AB|    3   11  11
                                   0| B|AC|AB|    4   11  12
                                      0|AJ|AC|   11   11  13
                                         0| A|    2   12  14
                                            0|    8   12  15
```

Main invariants and measures:

Symmetry	|V|	|R|	K	N	SVV	SV	SRV	HR	SR
0-symmetry	**15**	**105**	**15**	**105**	$\mathbf{1^{15}}$	*0*	$\mathbf{1^{105}}$	*2.0212*	***0***

Specified invariants and measures:

G	|E|	k	N^+	N^-	P	CL	MC	DM	SEV^+	SE^+	TRA	BRA
015-83	83	1	83	22	46	8	3	2	1^{83}	0	1.000	0

Comments: **a)** Graph ***015-83*** is ***2-distance-regular.*** **b)** Explicit clique sign no exist. **c)** Correspondingly to clique rule to open a pair graph with a large triangular value. Let it is pair graph $g_{7.15}$ with pair sign ***AT=+2.13.66.*** **d)** Processing results of *pair graph* $g_{7.15}$ of **015-83** in the form the *local sign matrix* $W_{7.15}$:

```
G:+2.7.20; H:+2.7.21; I:+2.8.26; J:+2.8.27; K:+2.8.28; L:+2.9.32;
M:+2.9.33; N:+2.9.34; O:+2.9.35; P:+2.10.41; Q:+2.11.48; R:+2.12.56

| 1  1  1| 2| 3| 4  4| 5| 6| 7| 8| 9|                                  Orb
| 2  7 15| 1| 9| 6 14|12| 5| 3| 4|11|   i   ABCDEFGHIJKLMNOPQR  deg    *
  0  R  R| P| K| Q  Q| L| Q| M| M| P|   2   000000000011200232   11    1
     0  R| P| K| Q  Q| L| Q| M| M| P|   7   000000000011200232   11    1
        0| P| K| Q  Q| L| Q| M| M| P|  15   000000000011200232   11    1
           0| K| O  O| H| P|-D|-D| N|   1   000200010010012400    9    2
              0| K  K|-F| K|-F|-E|-C|   9   001012000070000000    7    3
               | 0  Q|-B| P| J| J| N|   6   010000000210011140   10    4
                    0|-B| P| J| J| N|  14   010000000210011140   10    4
                       0| I| G| G| I|  12   020001212003000000    8    5
                          0|-A| I| N|   5   100000002010010330   10    6
                             0| I| I|   3   100101102200300000    8    7
                                0|-A|   4   100110102200300000    8    8
                                   0|  11   101000002000040300    7    9
```

e) There exist *explicit clique signs* ***H:+2.7.21*** and ***K:+2.8.28***. Consequently, in ***015-83*** exists ***7-clique*** and ***8-clique***. By corresponding adjacency lists B_{ij} of pair graphs recognize the intersected cliques:

v_i	1	2	5	6	7	9	11	12	14	15
7-clique	●	●	●	-	●	-	●	●	-	●
8-clique	●	●	●	●	●	●	-	-	●	●

f) The complement ***015-83C*** constitute a rare and with smaller cliques structure.

*

Example 34. Conclusive table of all there treated non-bisymmetric structures, arranged by symmetry measures ***SR***:

Nr	Notation	Exl	*SRV*	*SR*	*K*	*N*	Regularity	Parts	Commentary
1	**M20-30**	26	$10^1 30^2 60^2$	***0.7900***	1	5	*vg*	-	dodecahedra
2	**M20-160**						*vd*	-	complement
3	**M16-48**	24	$24^1 48^1$	***0.7796***	1	3	*vdgs*	-	Weisfeiler
4	**M16-72**						*vdcs*	-	complement
5	**M14-21**	23	$21^1 28^1 42^1$	***0.7655***	1	3	*vg*	2	Heawood
6	**M14-70**						*vdc*	-	complement
7	**M16-32**	25	$8^1 32^2 48^1$	***0.73985***	1	4	*vg*	2	hypercube
8	**M16-88**						*v*	-	complement
9	**P15-45**	27	$15^3 30^2$	***0.7267***	1	5	*vdc*	-	Kohov
10	**P15-60**						*vd*	-	complement
11	**M20-40**	29	$10^1 20^1 40^4$	***0.6300***	1	6	vg	5	orbitstructure
12	**P20-50a**	29	$10^1 20^5 40^2$	***0.6196***	1	8	*vgc*	-	Praust
13	**P20-120**						*vt*	-	complement
14	**P20-50b**	29	$10^1 20^9$	***0.5640***	1	10	*vgc*	-	Praust
15	**P24-36**	28	$12^{13} 24^5$	***0.4957***	1	18	*vg*	2	Tevet
16	**P24-240**						*vd*	-	complement
17	**L10-14**	30	$1^1 2^1 4^3 6^1 8^3$	***0.4593***	3	9	g	-	Petersen-sub
18	**L21-42**	31	$7^{12} 14^7 28^1$	***0.4590***	3	20	*vg*	-	Brinkman
19	**L21-168**						*dt*	-	complement
20	**L10-16**	30	$1^3 2^5 4^8$	***0.2994***	5	16	*d*	-	Petersen-sup
21	**L25-150**	32	$1^{20} 2^{128} 4^6$	***0.1290***	15	154	*vdst*	-	Weisfeiler
22	**L25-150c**						*v*	-	complement
23	***015-83***	33	1^{105}	***0***	15	105	*d*	-	Võhandu

CONCLUSION

It was story of „anew discover" of the graphs, where open the new relationships between structural attributes. On the other hand it is a practical processing and treatment mode of the graphs. We were demonstrated, that all the graphs, little and larges, have such attributes as orbits, adjacent- and sign structures. All the graphs are recognizable in constructive form with exactness up to isomorphism. We can to recognize *c* – clique-, *d* – distance-, *g* – girth-, *s* – strongly-, *v* – valence- regularity of graphs, etc.

The selection of *examples* with *comments* and *propositions* express there the processing results of algorithms, where give much attention to *symmetry properties,* particularly to bisymmetry of structure. The examples are selected so, that all the essential structural properties of symmetric and non-symmetric graphs are presented.

Structural approach differs at custom treatments in following:

- In structural approach has the word "structure" a sure meaning and import.
- Structural treatment of the graphs is concentrated to complete invariant of isomorphic graphs – to sign matrix ***W*,*** that is formed by simple initial data.
- Give up from complicated isomorphism testing, it be concealed in simple equivalence of sign matrices.
- Structure and all its attributes, such as paths, circuits, cliques, partition, orbits, symmetry, orbit and adjacent structures, structure systems etc are in a complex and completely recognizable by structural signs or their classes.
- Give up from treatment the symmetry properties by automorphism groups ***AutG***, it replaced with simple treatment the pair signs, that present the local isomorphisms of pair graphs.
- Given an exhaustive classification of symmetry kinds by orbits and their powers.
- Given up from treatment the reconstruction problems by ideology of Ulam conjecture, it is related with elementary changes, i.e. with reverse orbits of adjacent structures.
- A system of structures constitute an ordered complex of adjacent structures..

Do the results of 'structure semiotic' research to graph theory? To this question reply as a rule in this way that: *"... any new results about graphs could be regarded as graph theory as long as they shed new light on graphs. The substance is the most important. If you make generalizations just for the sake of generalizations, then most people might not find it too interesting"*.

But, for all that, what is structure semiotics? Is it a "structure philosophy" that takes to a "cognitional loneliness" or simply a "play on the graphs"? It is clear, that this "play" has a sense and is useful.

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